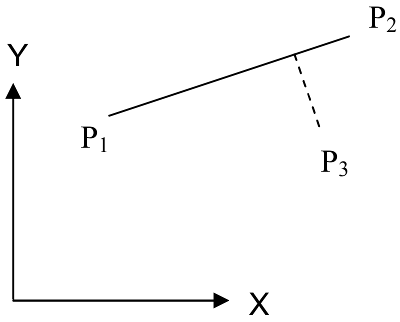
Distance between a point and a segment

The distance from point P_3 to segment defined by points P_1 and P_2 is calculated.

P_1: (x_1,y_1)
P_2: (x_2,y_2)
P_3: (x_3,y_3)

$$\lambda = \frac{(y_3 - y_1)(y_2 - y_1) + (x_3 - x_1)(x_2 - x_1)}{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$



Case $\lambda < 0$:

$$d = \sqrt{(x_1 - x_3)^2 + (y_1 - y_3)^2}$$

Case $\lambda > 1$:

$$d = \sqrt{(x_2 - x_3)^2 + (y_2 - y_3)^2}$$

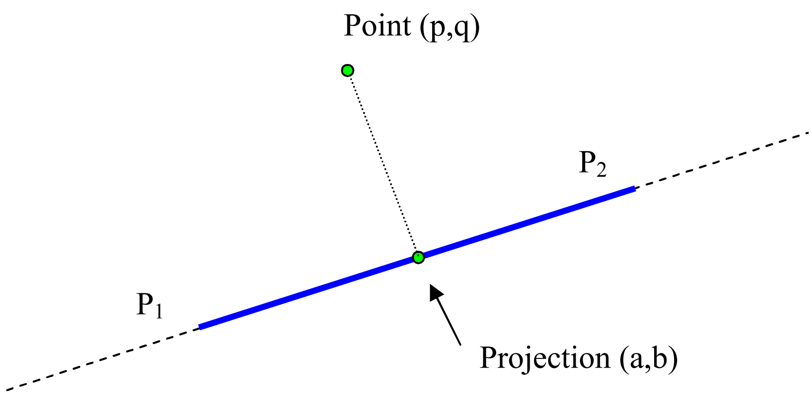
Case $0 <= \lambda <= 1$:

$$d = \sqrt{[x_1 + \lambda(x_2 - x_1) - x_3]^2 + [y_1 + \lambda(y_2 - y_1) - y_3]^2}$$

Projection of a point on the straight line defined by a segment

The coordinates (a,b) are calculated of the projection of point (p,q) on the straight line defined by segment $P_1(x_1,y_1)$-$P_2(x_2,y_2)$.

Equation of the straight line passing through P_1 and P_2:



$$y = k(x - x_1) + y_1$$

$$k = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

If $x_1 = x_2$ (vertical segment):

$a = x_1 = x_2$
$b = q$

If $x_1 \neq x_2$:

Equation of the straight line perpendicular to P_1P_2 passing through (p,q):

$$y = -\frac{1}{k}(x-p)+q$$

Coordinates of the projected point:

$$a = \frac{p+k(q-y_1)+k^2x_1}{1+k^2} \qquad b = y_1 + k(a-x_1)$$

Distance between two segments

Let one segment be defined by points P_1, P_2 and the other one by points P_3 and P_4. The corresponding carrier straight lines have these equations in vector form:

$\mathbf{P}_{12} = \mathbf{P}_1 + \lambda(\mathbf{P}_2 - \mathbf{P}_1)$
$\mathbf{P}_{34} = \mathbf{P}_3 + \mu(\mathbf{P}_4 - \mathbf{P}_3)$

where $\mathbf{P}_{12}$ and $\mathbf{P}_{34}$ represent vectors from the origin to points on the first and second straight lines respectively.

We first determine the distance between any two points $A(x_a,y_a)$ and $B(x_b,y_b)$ as a function of λ and μ. The 2 vector equations above can be written in matrix form as:

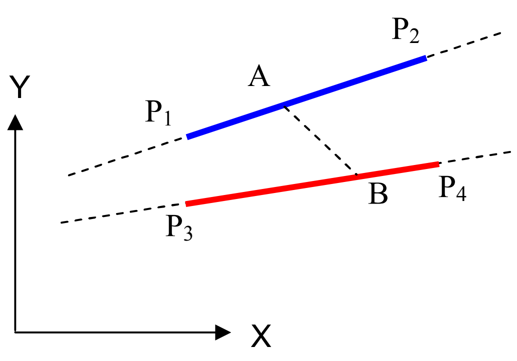
$$\begin{pmatrix} x_a \\ y_a \end{pmatrix} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} + \lambda \begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \end{pmatrix}$$
$$\begin{pmatrix} x_b \\ y_b \end{pmatrix} = \begin{pmatrix} x_3 \\ y_3 \end{pmatrix} + \lambda \begin{pmatrix} x_4 - x_3 \\ y_4 - y_3 \end{pmatrix}$$

The squared distance is:

$$D^2(A,B) = (x_a - x_b)^2 + (y_a - y_b)^2 =$$
$$[x_1 + \lambda(x_2 - x_1) - x_3 - \mu(x_4 - x_3)]^2 + [y_1 + \lambda(y_2 - y_1) - y_3 - \mu(y_4 - y_3)]^2$$

Now we determine for which values of λ and μ the distance is a minimum (i.e., 0, when the carrier lines intersect) by equating the derivatives to zero and solving for λ and μ:

$$\frac{\partial D^2}{\partial \lambda} = 0 \qquad \frac{\partial D^2}{\partial \mu} = 0$$



From these 2 equations we get:

$E\lambda - C\mu = P$
$C\lambda - F\mu = Q$

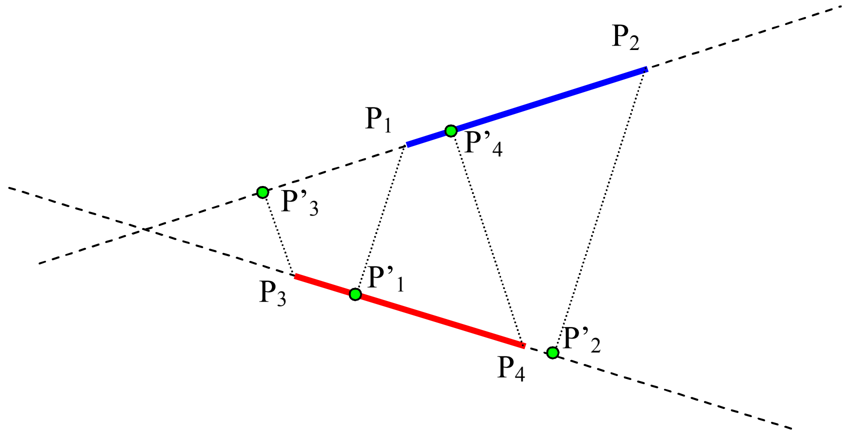
where:

$$E = (x_2 - x_1)^2 + (y_2 - y_1)^2$$
$$F = (x_4 - x_3)^2 + (y_4 - y_3)^2$$
$$C = (x_4 - x_3)(x_2 - x_1) + (y_4 - y_3)(y_2 - y_1)$$
$$P = (x_3 - x_1)(x_2 - x_1) + (y_3 - y_1)(y_2 - y_1)$$
$$Q = (x_4 - x_3)(x_3 - x_1) + (y_4 - y_3)(y_3 - y_1)$$

Solving for λ and μ:

$$\lambda = \frac{FP - CQ}{EF - C^2} \qquad \mu = \frac{PC - EQ}{EF - C^2}$$

If $0 \le \lambda, \mu \le 1$ the distance is 0 (intersection belongs to both segments). If $\lambda \notin [0,1]$ or $\mu \notin [0,1]$ or EF = C^2 the projections must be found of each segment over the other segment, P'_1, P'_2 and P'_3, P'_4.



If (p,q) represent the coordinates from the point to be projected and (a,b) represent the coordinates of its projection, the distance between these two points is:

$$d = \sqrt{(a-p)^2 + (b-q)^2}$$

If $a \notin [x_k, x_{k+1}]$ where x_k and x_{k+1} represent the coordinates x from the segment in the other straight line, this distance is discarded.

Finally, the distance between the segments is:

D = Min $\{d_i\}$

where $\{d_i\}$ = {d(P_1P_3), d(P_1P_4), d(P_2P_3), d(P_2P_4), d($P_1P'_1$), d($P_2P'_2$), d($P_3P'_3$), d($P_4P'_4$)}

and d(P_iP_j) is the distance between points P_i and P_j