Under the current definition of a Magic Hexagon, there is only One Magic Hexagon of a Non-Trivial Size, and that one uses the complete range of integers inclusively from 1 thru 19, to add to the magic Number 38. Indeed, a Gentleman by the name of C.W. Trigg, proved that there exists only 1 Magic Hex of Any Size that starts with the Number One. This came about after Martin Gardner passed a sketch of such a Magic Hex to C.W. Trigg, having himself received it from Clifford W. Adams, who spent from 1910 to 1957 developing this arrangement. {Like any true Mathematical Adventurer, he spent the next 5 years trying to find it again, after having misplaced the sheet of paper he had sketched the solution upon.} [1]

Unfortunately, it seems that there has been little, if any, development into Non-Normal Magical Hexes, or Magical Hexagons that uses a Range of numbers k, k+1, k+2, … k+C – 1, where C denotes the number of cells in the Hexagonal structure, whereby every linear Row adds to the same Magical number. I believe this lack of development is due to the Definition of a Magic Hexagon being identical to the case of a Normal Magic Hexagon, instead of the Normal Magic hexagon being a special Case of a general Magical Hexagon starting with k.

So I would like to make the case that, unlike Non-Normal Magic Squares being just a linear transformation of a Pure Magic Square that starts with 1, that if a Magical Hexagon starts with a number other than 1, it is distinct and should not be considered trivial. Indeed, Non-Normal Magical Hexagons have mathematical properties which extend beyond Mr. Triggs proof.

Mr. Trigg showed that for a Magic Hexagon that starts with 1 to exist, the Number 5 must be wholly divisible by 2n – 1, where n is the number of "Rings" in any given hexagonal arrangement of close packed hexagons is certainly true. However this relation was developed specifically for the case where a Magic Hexagon starts with the Number 1. It can be shown that the More general case is the Following:

If One was to denote the Number of Rows on any side in a Magic Hexagon as c_Rows, and that the magic Hexagon has c_R rings to it, such that the center Ring contains 1 cell, the 2nd ring contains 6, the Third, 12, and so on, where by the total number of cells, denoted as c_C, equals $3*c\_R^2 – 3*c\_R + 1$, and that the Hexagonal structure contains the integral numbers LowNum, LowNum+1, LowNum+2,… HighNum Inclusively, Then for the structure to have a Magic Number, (8* LowNum – 3)/c_Rows must be a whole number.

Furthur, it can easily be shown that the Following Formulas are Valid for All c_R Ring Magical Hexagons of Any size:

"For some r:

*a_N = r*c_Rows*
*mNum = r*c_C*
*LowNum = r*c_Rows – (c_C – 1)/2*
*HighNum = r*c_Rows + (c_C – 1)/2*
*LowNum = r*c_Rows – 3*(c_Rows² - 1)/8*
c_Rows = [4r ⁺/₋ u]/3, where u² = 16r² – 3{8*LowNum – 3}, or:*
(4r – u)(4r + u) = 3{8*LowNum – 3}*
where a_N is the average number, and mNum is the Magic number."

Finally, there exists a Total of 56 Magical Hexagons with 3 Rings:

The Original, Starts with 1, ends with 19, adds to 38
36 That start with –4 and ends with 14, adding to 19, and
19 that start with –9 and ends with 9, whose Magic is 0.

There are Millions of 4 Ring and 5 Ring Magical Hexagons. And I see no reason to disbelieve that there exists Countless Other, n Ring Magical Hexagons.

-Louis Hoelbling
-February 28, 2004

First, some Basic Equations:

Figure 1 illustrates the Rings of a Basic Hexagonal structure.
The Center Cell is considered to be Ring 0.
The 6 Cells surrounding the Center Cell is Ring 1.
The 12 Cells surrounding Ring 1 is Ring 2.
As you will Notice, the Number of cells in Ring N is 6N.

So, If we were to Denote the Number of Rings in such a Hexagonal Structure to be **c_R**, and the Total number of cells in a Hexagon as **c_Cells**, then c_Cells in:

A 1 Ring Hex is:	1
A 2 Ring Hex is:	1 + 6 = 7
A 3 Ring Hex is:	7 + 12 = 19
A 4 Ring Hex is:	19 + 18 = 37

...

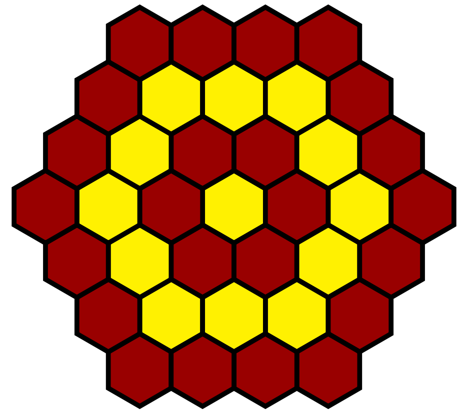

Figure 1: The Rings of a Hex

This Numerical Progression can be processed using Newtons Method of Differences:

c_R:	1		2		3		4
c_Cells:	1		7		19		37
Diff_0:		6		12		18	
Diff_1:			6		6		

If we assumed that there existed a Hexagonal structure with 0 Rings, which would provide a backwards continuity with the observed uniformity with the Diff_1 row, We'd end up with the Following:

c_R:	0		1		2		3		4
c_Cells:	1		1		7		19		37
Diff_0:		0		6		12		18	
Diff_1:			6		6		6		

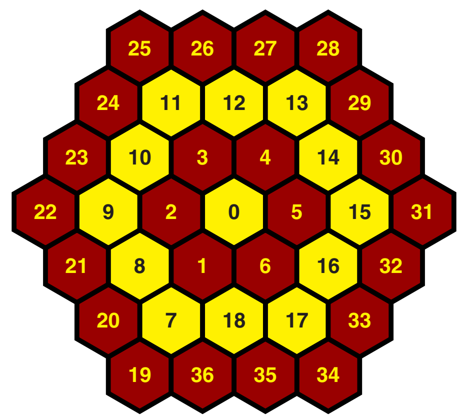

Figure 2: The Cells of a Hex

Which allows us to create the relation:

c_Cells = 1 + 0*c_R + 6*c_R(c_R - 1)/2

Or:

$$c\_Cells = 3c\_R^2 - 3c\_R + 1$$

And, Double checking against the 37 Cells, (0 thru 36) in the 4 Ring Hex of Figure two, 37 is Indeed = 3*(4^2) - 3*4 + 1

c_Rows, or the Count of rows on a "Side" of a Hexagonal Structure, can be expressed as c_Rows = 2*c_R - 1. From Figure 3, we see that a 1 Ring Hexagon has 1 Row, as repre sented by Line A. A 2 Ring Hexagon contains the rows B_1 thru B_2, a Total of 3. A 3 Ring contains the Rows C_1 thru C_2 {5}, and a 4 Ring contains the rows D_1 thru D_2 {7}. This Linear Expression: 1, 3, 5, 7 in relation to their individual c_R's: 1,2,3,4, is easily seen to produce the relation:

c_Rows = 2*c_R - 1

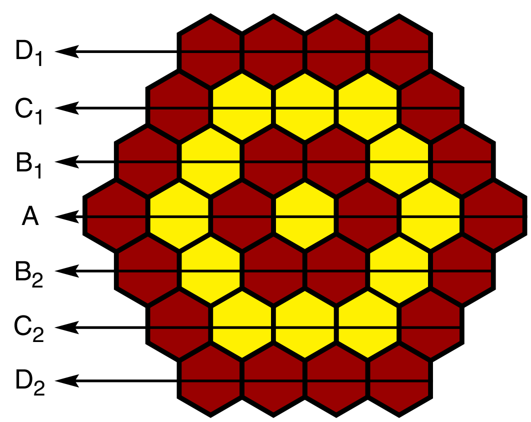

Figure 3: The Rows of a Hex

And, if the Hexoganal Structure contained a continuous range of integers, which sequentially incrimented by 1 from one inte ger to the next, this range could be said as starting with **LowNum**, and ending with **HighNum**. If we were to denote the average number in this range as **c_A**, then simply:

c_A = (LowNum + HighNum)/2

Also, concerning the Numbers LowNum and HighNum, since it is a continuous range, and the number of integers in the range LowNum thru HighNum must be equal to the number of cells in the Hexagonal Structure, c_C, then:

c_C = HighNum – LowNum + 1

Which, upon Manipulation, we see that:

HighNum = LowNum + c_C – 1

The Magic number, or **mNum**, is the number that each row adds to. Every row must add to the same number, and so the sum of all the cells taken together must split evenly between the number of rows on a side. So, the complete range of numbers from LowNum to HighNum, since there are c_C of them, and they average to c_A, adds to c_A*c_C. Therefore, if that total sum is evenly divided by c_Rows, then:

mNum = c_A*c_C/c_Rows

<u>The Terminology So Far:</u>

c_R	Number of Rings in the Hex. A hex with 7 cells: c_R = 2, 19: c_R = 3
c_C	Number of cells in a Hex.
c_Rows	Number of rows along 1 side of a Hex.
LowNum	Lowest number used in Range of numbers used.
HighNum	Highest Used number in range of numbers used.
mNum	The Value that all cells in a row sum together to, or “The Magic Number”.
a_N	The Average Number, or (LowNum + HighNum)/2

<u>The Basic Equations, or Identities:</u>

$c\_C = 3*c\_R^2 - 3*c\_R + 1$ $\quad\quad$ $c\_R > 0$

HighNum = LowNum + c_C – 1

c_Rows = 2*c_R – 1

a_N = (LowNum + HighNum)/2,

mNum = a_N * c_C /c_Rows

<u>The Basic Equations reordered, or rexpressed with substitutions:</u>

c_R = (c_Rows + 1)/2 $\quad\quad$ *Substituting this into the identity for c_C, we see:*

$c\_C = (3*c\_Rows^2+1)/4$

a_N = LowNum + (c_C – 1)/2 $\quad\quad$ *Substituted the HighNum Identity. Reordered we see:*

LowNum = a_N – (c_C – 1)/2

<u>The Following will be explained on the next few pages:</u>

$mNum = (9*c\_Rows^4 + 6*c\_Rows^2*(4*LowNum - 1) + 8*LowNum - 3)/32*c\_Rows$

For a Magical Hex to start with LowNum, then:

(8* LowNum – 3) must be divisible by c_Rows

For some r:

a_N = r*c_Rows

mNum = r*c_C

LowNum = r*c_Rows – (c_C – 1)/2

HighNum = r*c_Rows + (c_C – 1)/2

$LowNum = r*c\_Rows - 3*(c\_Rows^2 - 1)/8$

$c\_Rows = [4r \ {}^{+}\!/_{-}\ u]/3$, where $u^2 = 16r^2 - 3*\{8*LowNum - 3\}$, or:

(4r – u)(4r + u) = 3*{8*LowNum – 3}

How did you get: mNum = (9*c_Rows^4 + 6*c_Rows^2*(4* LowNum –1) +8* LowNum –3)/32*c_Rows?

Starting with the equation: mNum = a_N * c_C /c_Rows

Substituting a_N = LowNum + (c_C – 1)/2 and c_C = (3*c_Rows^2+1)/4, you can knead it into:

mNum = (9*c_Rows^4 + 6*c_Rows^2*(4* LowNum – 1) +8* LowNum – 3)/32*c_Rows

with the Following:

mNum = a_N * c_C /c_Rows
mNum = (LowNum + (c_C – 1)/2)*c_C /c_Rows
mNum = (LowNum + (((3*c_Rows^2+1)/4) – 1)/2)*((3*c_Rows^2+1)/4)/c_Rows
= (LowNum + (((3*c_Rows^2+1)) – 4)/8)*((3*c_Rows^2+1)/4)/c_Rows
= (LowNum + (3*c_Rows^2+1 – 4)/8)*((3*c_Rows^2+1)/4)/c_Rows
= ((8*LowNum + 3*c_Rows^2 – 3)/8)*((3*c_Rows^2+1)/4)/c_Rows
= (8*LowNum+ 3*c_Rows^2– 3)*(3*c_Rows^2+1)/32*c_Rows
= ((24*LowNum*c_Rows^2+8*LowNum)+(9*c_Rows^4+3*c_Rows^2)+(–9*c_Rows^2–3))/32*c_Rows
= (9*c_Rows^4+24*LowNum*c_Rows^2+(3*c_Rows^2–9*c_Rows^2) +8*LowNum–3)/32*c_Rows
= (9*c_Rows^4+24*LowNum*c_Rows^2–6*c_Rows^2+8*LowNum–3)/32*c_Rows
mNum = (9*c_Rows^4 + 6*c_Rows^2(4*LowNum – 1) + 8*LowNum – 3)/32*c_Rows

What is the significance of: (8* LowNum –3) must be divisible by c_Rows?

By inspecting the preceeding mNum equation, we see that every term in the numerator is a multiple of c_Rows, except for the portion (8*LowNum – 3). So, for mNum to be a whole number, then (8*LowNum – 3) must be wholly divisible by c_Rows. This is also equivalent to Mr. Triggs requirement of 5 being divisible by 2n – 1, when LowNum = 1. To see this, lets examine C. W. Triggs Proof.

He started with the target of finding the equation that produces the Magic Number of a Normal Hexagon. This generic Equation proved to be:

mNum = [9*(n^4 – 2n^3 + 2n^2 – n) + 2]/(2*(2n – 1))

“which requires 5/(2n-1) to be an integer for a solution to exist. But this is an integer for only *n* = 1 (the trivial case of a single hexagon) and Adams's *n* = 3 (Gardner 1984, p. 24).” [2]

To See how it requires 5 to be divisible by 2n – 1, Lets redo the equation with the identity k = 2n – 1, by substituting n =[(k+1)/2]:

mNum = [9***{**[(k+1)/2]^4 – 2[(k+1)/2]^3 + 2[(k+1)/2]^2 – [(k+1)/2]**}** + 2]/(2k)
mNum = [9***{**[((k+1)]^4/16) – 2([(k+1)]^3/8) + 2([(k+1)]^2/4)– [(k+1)/2]**}** + 2]/(2k)
mNum = [9***{**[((k+1)]^4/16) – ([(k+1)]^3/4) + ([(k+1)]^2/2)– [(k+1)/2]**}** + 2]/(2k)

Multiply by 16/16

mNum = [9***{**[k+1]^4 – 4[k+1]^3 + 8[k+1]^2 – 8[k+1]**}** + 32]/(32k)

= [9***{**k^4	+4k^3+	6k^2	+4k	+1
	-4k^3	-12k^2	-12k	-4
		+8k^2	+16k	+8
			-8k	-8**}**+32]/(32k)

= [9***{**k^4+2k^2– 3**}** + 32]/(32k)
= [9***{**k^4+2k^2**}** – 27 + 32]/(32k)
= [9***{**k^4+2k^2**}** + 5]/(32k)

And, as you can see, the numerator is completely divisible by k except for 5. So, for mNum to be whole, 5 must be divisible by k, or 2n – 1.

This relation, 5/(2n – 1) can be seen to be equivalent to my relation, (8* LowNum – 3)/c_Rows, when lowNum = 1, and using the identity c_Rows = 2*c_R – 1, if we say c_R = n. The relation does show that there is only 1 significant mHex that starts with 1, because it has to be a 3 Ring Hex {n = 3}, and there is only 1 solution to a 3 Ring hex when 0 < LowNum, and that is when LowNum = 1. However, there are More 3 Ring Magical Hexagons if we are allowed to start with a number other than One.

If we look at (8* LowNum – 3)/c_Rows, since for a 3 Ring whose c_Rows is Always 5, any LowNum that satisfies the Whole Divisibility of 8*LowNum – 3 by 5 can be a possible start for a magic hex. Now, we know that LowNum can be 1, but it can also be seen that when LowNum = 5s + 1, for some whole s also allows 8*LowNum – 3 to remain divisible by 5, since by substitution, 8*(5s + 1) – 3 is offset from 8*1 – 3 by a whole multple of 5. Thus, you could have a LowNum = -4, when s = -1, or –9 when s = -2. In Actuality there are 36 Distinct 3 Ring Magical Hexes that start with –4, whose mNum = 19, and 19 Three Ring Magical Hexes that start with –9 whose mNum = 0. Not Counting Rotations, Reflections, Multiplicative Transformations, or any combination thereof.

This then brings me to my delemma. Why should a distinction be made that a Magic Hex must start with 1? Certainly, One could argue that in the world of Magic Squares, All Magic Squares start with 1, and the range of numbers used are continuous and step from one to the next by 1. If it starts with a number other than 1, but still incrimentally steps from one number to the next by 1, then it is just a linear transformation of a Parent Square that starts with 1. Magic Squares are Linear in Nature, ie.. if you Have a Magic Square that uses a continuous Range of Numbers from LowNum to HighNum, then it can be directly remapped to another Magic Square using the number Range LowNum + k to HighNum + k, where its mNum remaps to mNum + k*Order, where Order is the size of the Magic Square taken from the count of the cells on its side. This is done by adding k to each number in the Square, and the resultant Magic Square is not, IMHO, Unique, but is only a variation of its original Parent Magic Square.

Now, with Magical Hexes, there Can be no linear transformation by simply adding k to each cell in a magic hex, for the following reason. The Result is not Magical! For example, the familiar 3 Ring 19 cell Magic Hex contains 3 cells in its external rows. If you Add k to each value in that Row, the mNum of that row increases to mNum + 3k. Now, the rest of the Hexagon, for it to remain Magical, should also have mNum + 3k as the rowsum. However, if you look at the row immediately beneath an external row, it contains 4 cells, and if you've added k to each individual cell throughout the Magic Hex, then that row of four now adds to mNum + 4k, which does not equal the external row above it. So, Magical Hexes cannot be linearly transformed.

This means that assuming if a Magical hex exists that starts with the number 3, that it is linearly equivalent to some "Parent" Magic Hex that started with 1, and is then trivial and can be considered sub standard, is wrong. If a mHex starts with any number <> 1, with a continuous range of numbers k, k+1, k+2, …, then it is linearly Distinct from and is as valid as any magic hex that starts with 1.

<u>Whats THIS All About!!!</u>

"For some r:

$a\_N = r*c\_Rows$

$mNum = r*c\_C$

$LowNum = r*c\_Rows - (c\_C - 1)/2$

$HighNum = r*c\_Rows + (c\_C - 1)/2$

$LowNum = r*c\_Rows - 3*(c\_Rows^2 - 1)/8$

$c\_Rows = [4r\ {}^{+}/_{-}\ u]/3$, *where* $u^2 = 16r^2 - 3*\{8*LowNum - 3\}$, *or:*

$(4r - u)(4r + u) = 3*\{8*LowNum - 3\}$*"*

Glad you Asked! The Equation for mNum,

$$mNum = (9*c\_Rows^4 + 6*c\_Rows^2*(4*LowNum - 1) + 8*LowNum - 3)/32*c\_Rows$$

I feel is not as elegant as it could be, and using the divisibility test on 8* LowNum – 3 I feel lacks a certain deterministic flavor. So:

Considering the identity: $c\_C = (3*c\_Rows^2+1)/4$

It is obvious that c_C and c_Rows are relative primes, that is, they share absolutely no common divisor whatsoever, except for 1.

Let me Quote sql_lall when he helpfully pointed out the following:

> quote:
> *originally posted by sql_lall, at http://www.vbforums.com/showthread.php?s=&postid=1633305#post1633305*
> **Let X = c_Rows (make it cleaner)**
>
> **using the fact that gcd(a,b) <= gcd(Ya, b), and <= gcd(a^2, b)**
>
> **gcd((3*X^2+1)/4, X) <= gcd(3X^2+1, X) <= gcd(3X^2+1, 3X^2)**
>
> **of course, gcd(p, p+1) = 1**
> **so, gcd(3X^2+1, 3X^2) = 1**
> **so, gcd((3*X^2+1)/4, X) <= 1**
>
> **of course, it can't be smaller, so it must = 1 !!**

Or, More Simply:

> *Bugzpodder Points out, same thread:*
> **(3*x^2+1)/4 and x**
> **suppose some prime p divides both numbers, then p|x (p divides x) and p|3x^2+1**
> **but since p|x, then p|3x^2, hence p|1, so p=1**

Therefore, since c_C and c_Rows are relative primes, and considering mNum must be a whole number, then in

mNum = a_N*c_C/c_Rows

a_N Must be a whole multiple of c_Rows, so lets express a_N as:

a_N = r*c_Rows.

Doing so, we see that, upon substitution, that all Magic Numbers, or the sums that a row must add to, is:

mNum = r*c_Rows*c_C/c_Rows

Or:

mNum = r*c_C

for some r.

The Following 2 Equations are actually nothing new, just previous equations with the above two identities substituted in:

LowNum = r*c_Rows – (c_C – 1)/2
HighNum = r*c_Rows + (c_C – 1)/2

And this last is the LowNum equation, above, with c_C = $(3\text{*c\_Rows}^2+1)/4$, substituted in:

LowNum = r*c_Rows – 3*$(\text{c\_Rows}^2 - 1)/8$

__**Wait! Aren't you skipping:**__

c_Rows = [4r $^+/_-$ u]/3, where $u^2 = 16r^2 - 3*\{8*\text{LowNum} - 3\}$, or:
(4r – u)(4r + u) = 3{8*LowNum – 3}"*

No, I'm just seperating this from the last because of its significance.
If you balance out the last equation of the last section:

LowNum = r*c_Rows – 3*$(\text{c\_Rows}^2 - 1)/8$

To this:

$3*\text{c\_Rows}^2 - 8*r*\text{c\_Rows} + 8*\text{LowNum} - 3 = 0$

Then solving the Quadratic for c_Rows, you get:

c_Rows = [4*r $^+/_-$ Sqr($16r^2 - 3*\{8*\text{LowNum} - 3\}$)]/3

Now, saying $u^2 = 16r^2 - 3*\{8*\text{LowNum} - 3\}$,we see that, for c_Rows to be whole,

$16r^2 - u^2 = 3*\{8*\text{LowNum} - 3\}$

Or:

$$(4r-u)(4r+u) = 3*\{8*LowNum -3\}$$

and so c_Rows = $[4*r \ {}^{+}/_{-} \ Sqr(16r^2 - 3*\{8*LowNum - 3\})]/3$ can then be expressed as:

$$c\_Rows = [4*r \ {}^{+}/_{-} \ u]/3$$

Which requires $[4*r \ {}^{+}/_{-} \ u]$ to be wholly divisible by 3.

This allows you to easily deterimine the Ring Count and RowCount of som Hex if you wanted to find potential Hexes that start with some LowNum.

So, lets revisit the case where LowNum = 1, just to see how this works. We see that there must be some {r,u} such that:

$$(4r-u)(4r+u) = 3*\{8*1 - 3\}$$

or

$$(4r-u)(4r+u) = 15$$

Now, the distinct factor pairs of 15 are 1*15, 3*5
Which gives us 2 cases to consider:

<u>Case #1:</u> 1*15:

4r-u = 1
4r+u = 15

As you can see, 15 is divisible by three, so the sub case, where 4r+u = 15 is a potential solution:
c_Rows =[4*r + u]/3 = 15/3 = 5
And, since c_R = (c_Rows + 1)/2, then c_R = 3.

<u>Case #2:</u> 3*5:

4r-u = 3
4r+u = 5

As you can see, 3 is divisible by three, so the sub case, where 4r-u = 3 is a potential solution:
c_Rows =[4*r – u]/3 = 3/3 = 1
And, since c_R = (c_Rows + 1)/2, then c_R = 1.

Therefore, there exists 2 sizes of Hexagons that can produce an mHex starting with 1, a 1 Ring Hex and a 3 Ring Hex. The 1 Ring Hex, with 1 Cell is trivial, so the 3 Ring hex, containing c_C = $(3*c\_Rows^2+1)/4 = (3*5^2 + 1)/4 = 76/4 = 19$ cells, Is the Well Known Magic Hex that adds to 38, using the numbers 1 thru 19 inclusive.

Now, Are there other magic sums that can produce more 3 Ring Magic Hexes, other than the one that starts With 1? Going back to LowNum = r*c_Rows – (c_C – 1)/2, for a 3 Ring Hex, we know that c_C = 19, c_Rows = 5, so:

LowNum = 5r – 9

So, If we check for the cases 0 <= r < 2, { we already know LowNum = 1 when r = 2}, then LowNum can be –9 or –4, and with mNum = r*c_C, we see that mNum = 0 when r = 0, and mNum = 19 when r = 1.

This implies that there are more Magical 3 Ring Hexes than just the Already known one, that don't start with 1. And indeed, When r = 0, there are 19 Unique Magic 3 Ring Hexes, and When r = 1, there are 36 Unique 3 Ring mHexes, excluding their rotations, reflections, and negations.

In Total, there are 56 three Ring Magical Hexes, and no more. No Other Magial Hexes of Order 3 can be made when r > 2.

<u>So:</u>

If one were to build a Magic c_R Ring Hex, the Following Formulas would be helpful:

Number of Cells, c_C in a c_R Ring Hex:	$c\_C = 3*c\_R^2 - 3*c\_R + 1$
Number of Rows, c_Rows in c_R Ring Hex:	c_Rows = 2*c_R – 1

if we say that the Level of c_R Ring Magic Hexagon to Solve For is r:

Magic Number that a c_R mHex, Level r Adds to:	mNum = r*c_C
Min Number used in its Cells:	LowNum = r*c_Rows – (c_C – 1)/2
Max Number Used:	HighNum = r*c_Rows + (c_C – 1)/2

In Conclusion, with these pages, and the Magical 5 Ring and 6 Ring hex, as seen on the Cover, along with the Following samples of Non-Normal Magical Hexes, I believe this shows that there is a Richer world of Magical Hexagons out there, that Magical Hexagons are as diverse and unlimited as the world of Magic Squares, if we don’t start with the number One.

- Louis Hoelbling
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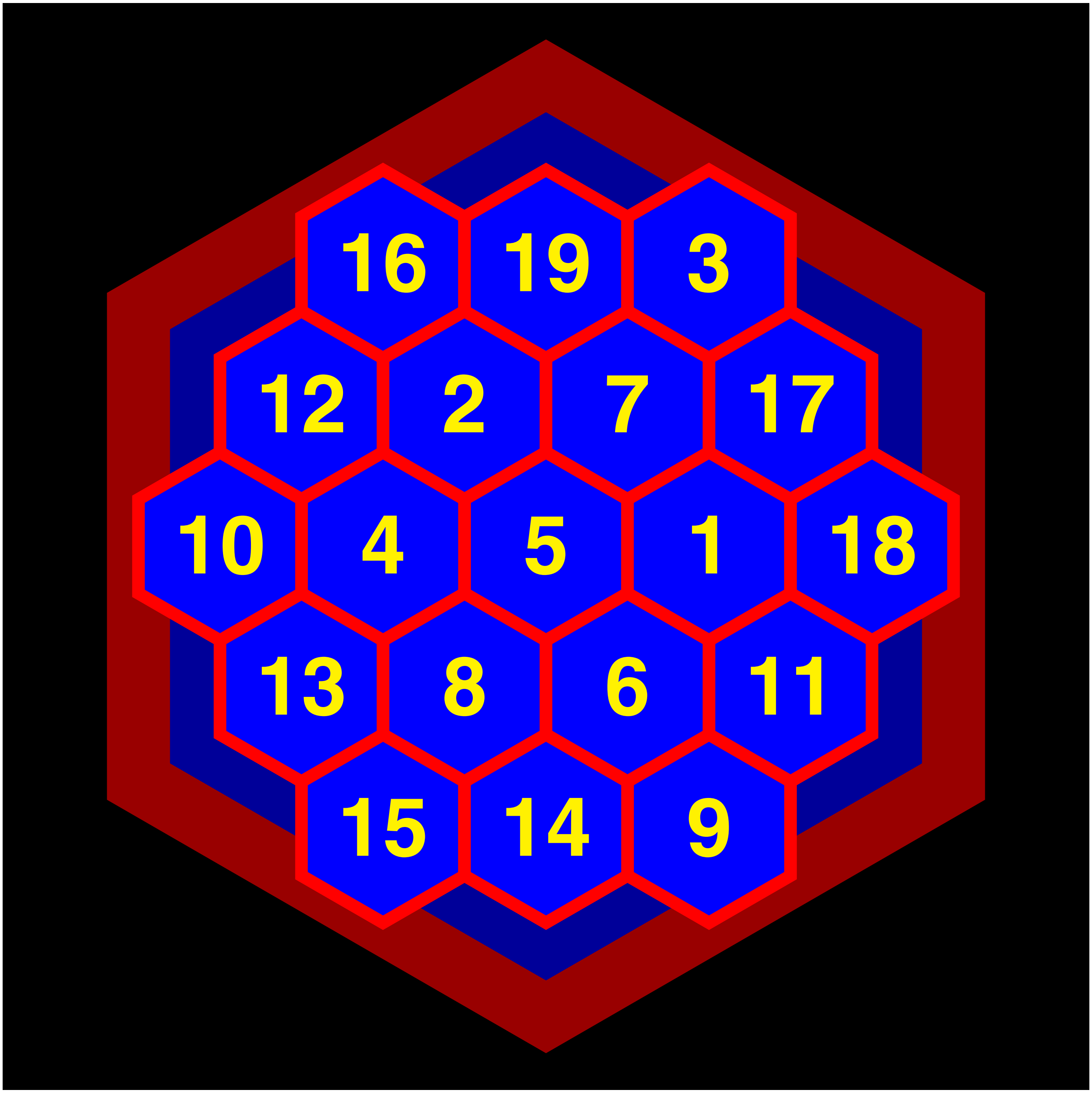
- Phone: (585) 288-1240

References:

1) Gardner, M. "Permutations and Paradoxes in Combinatorial Mathematics." *Sci. Amer.* **209**, 112-119, Aug. 1963.

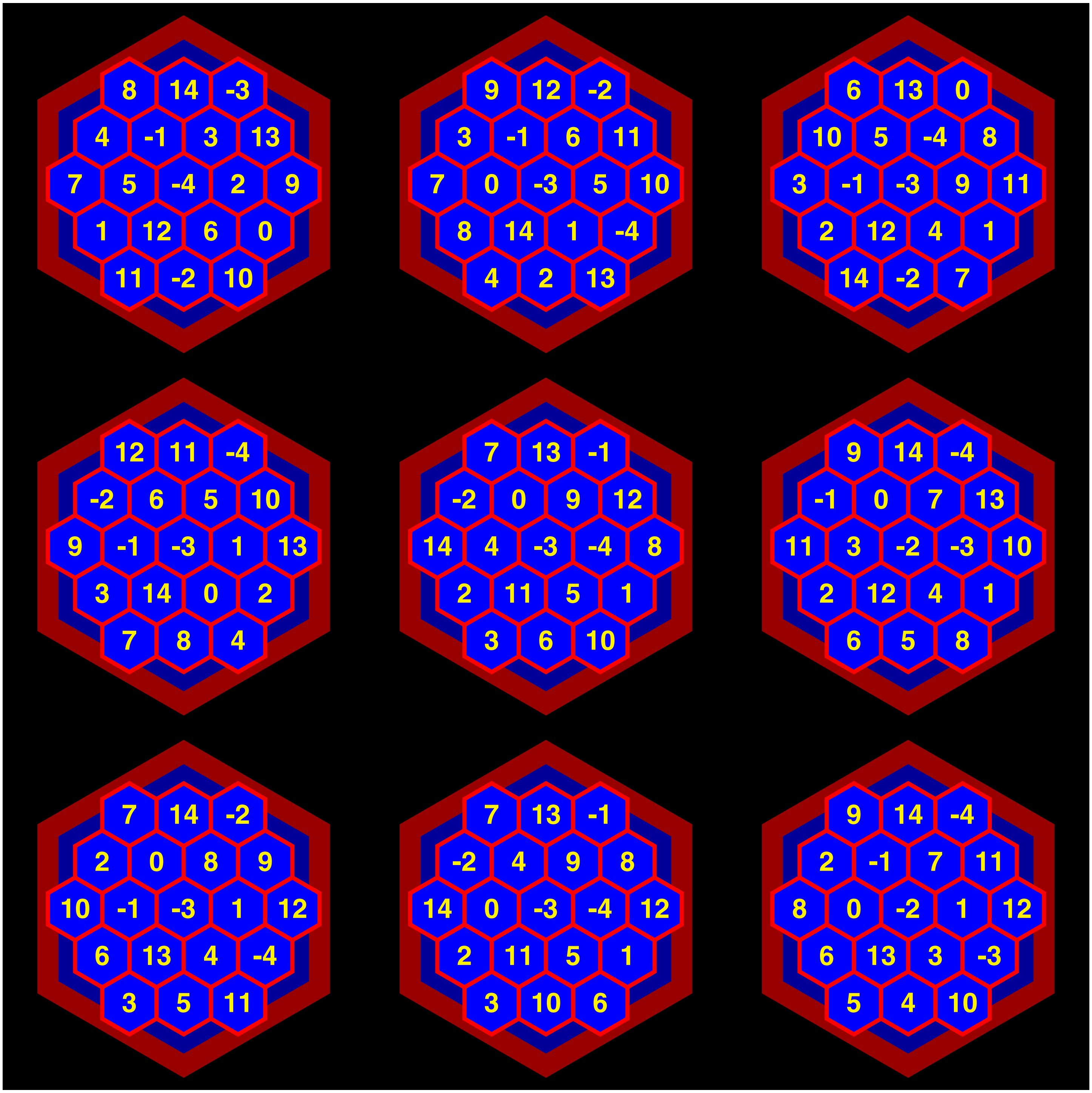
2) Eric W. Weisstein. "Magic Hexagon." From *MathWorld*--A Wolfram Web Resource. http://mathworld.wolfram.com/MagicHexagon.html

This collection of Magical Hexagons would not be complete if I were to not include a rendition of the Original "Magic Hexagon".



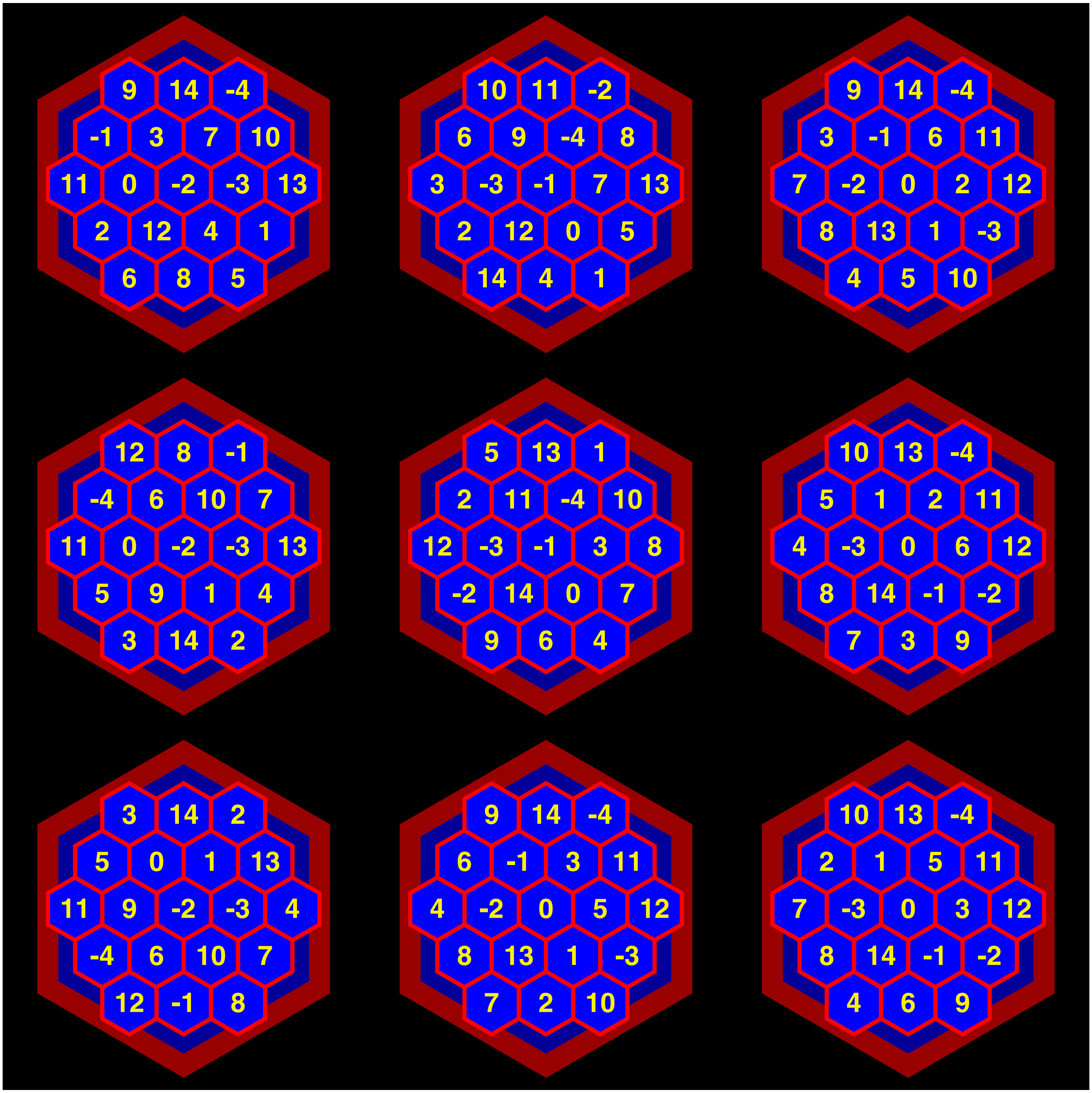
Vital Statistics:
c_R = 3
r = 2
LowNum = 1
HighNum = 19
mNum = 38

ADDS TO 19



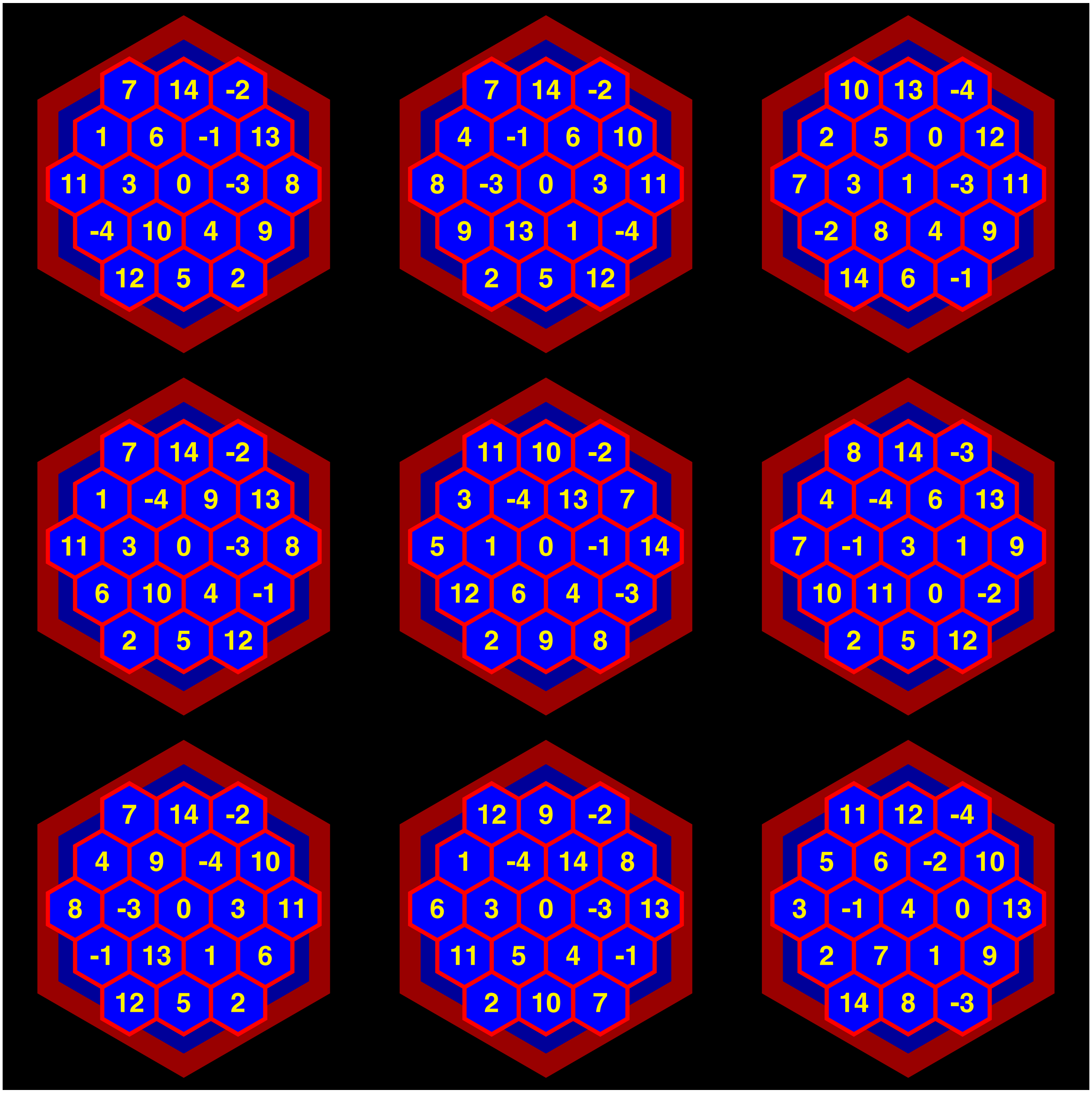
Vital Statistics:
c_R = 3
r = 1
LowNum = -4
HighNum = 14
mNum = 19

ADDS TO 19



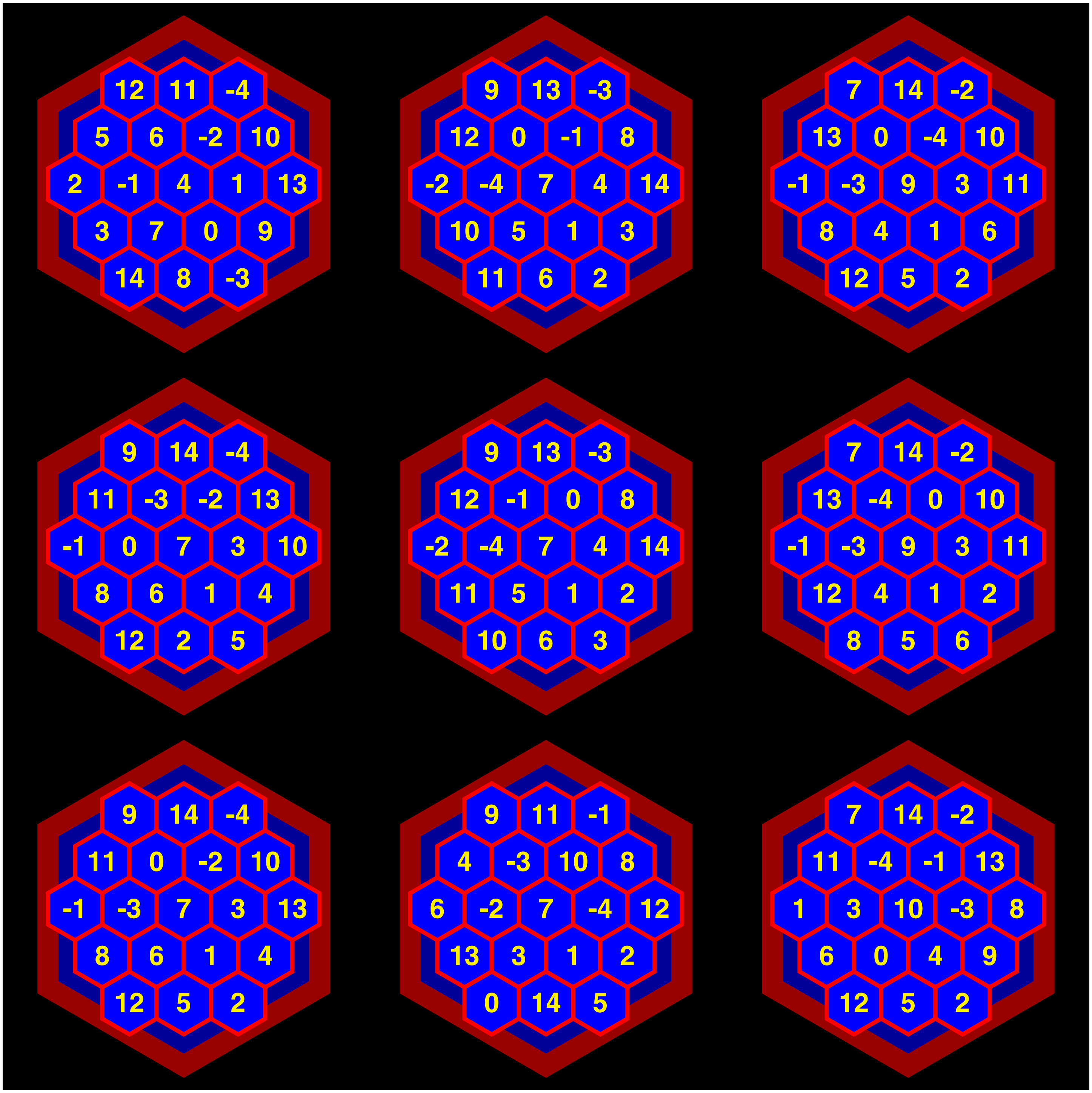
Vital Statistics:
c_R = 3
r = 1
LowNum = -4
HighNum = 14
mNum = 19

ADDS TO 19



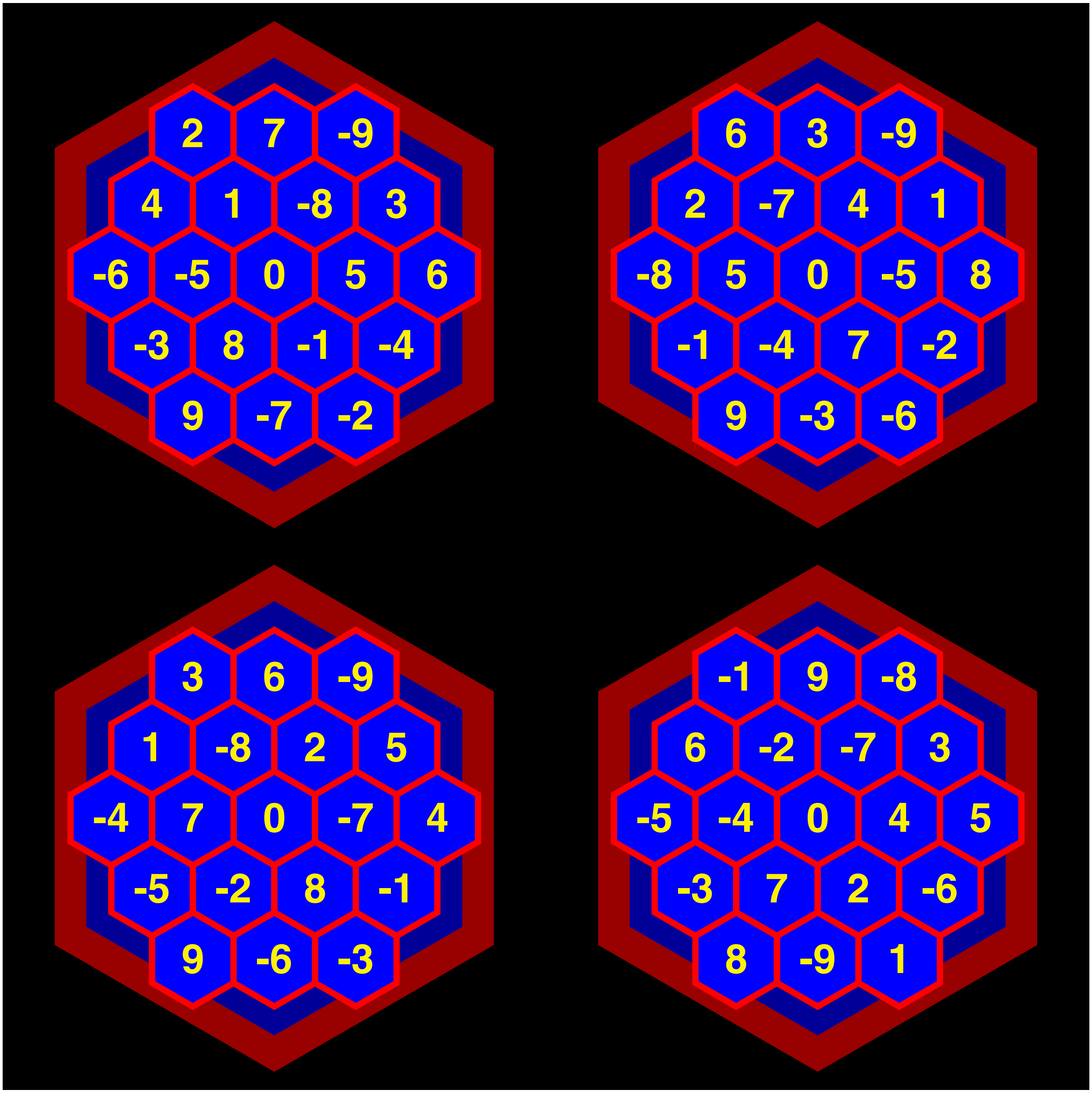
Vital Statistics:
c_R = 3
r = 1
LowNum = -4
HighNum = 14
mNum = 19

ADDS TO 19



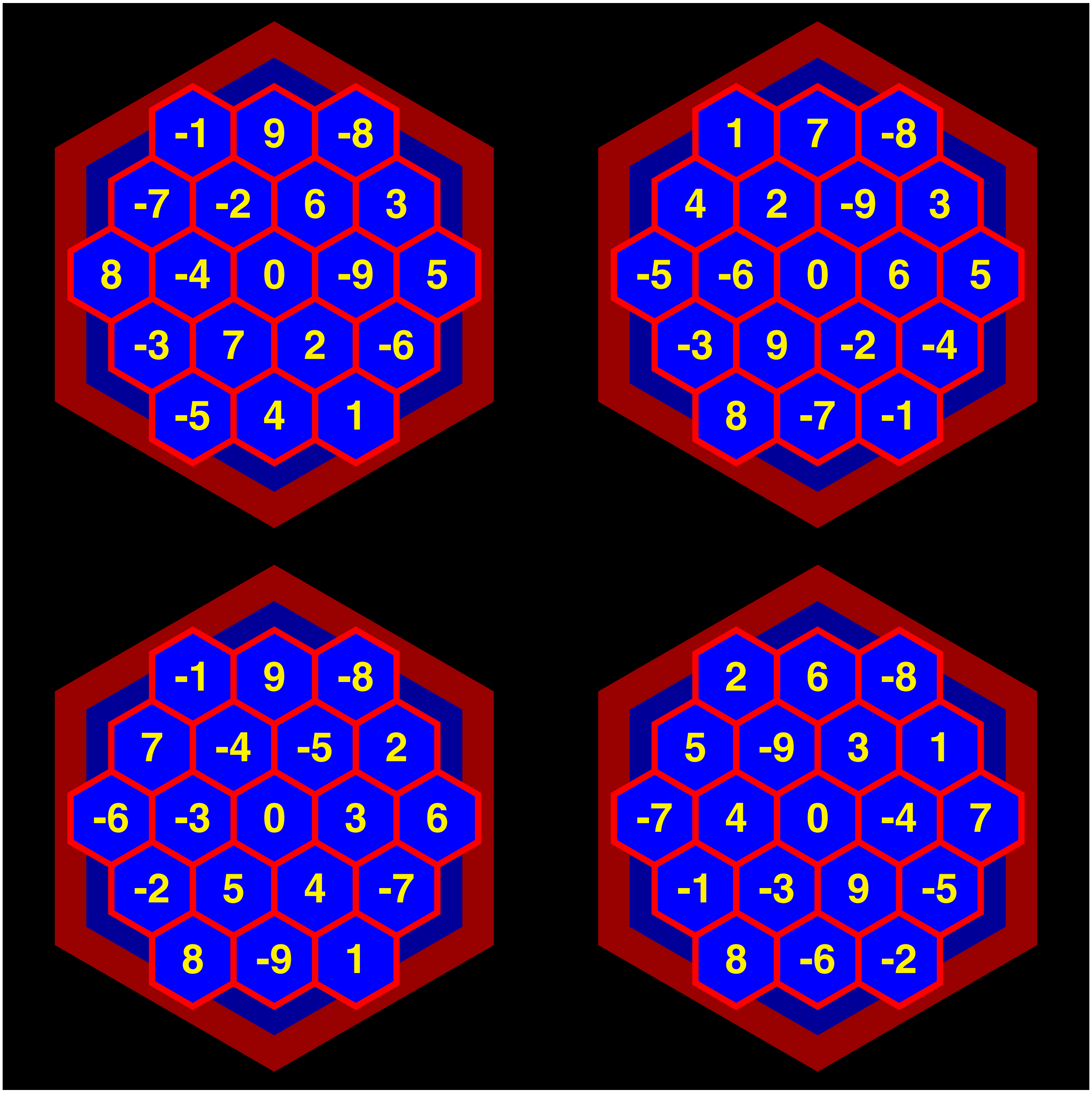
Vital Statistics:
c_R = 3
r = 1
LowNum = -4
HighNum = 14
mNum = 19

ADDS TO 0



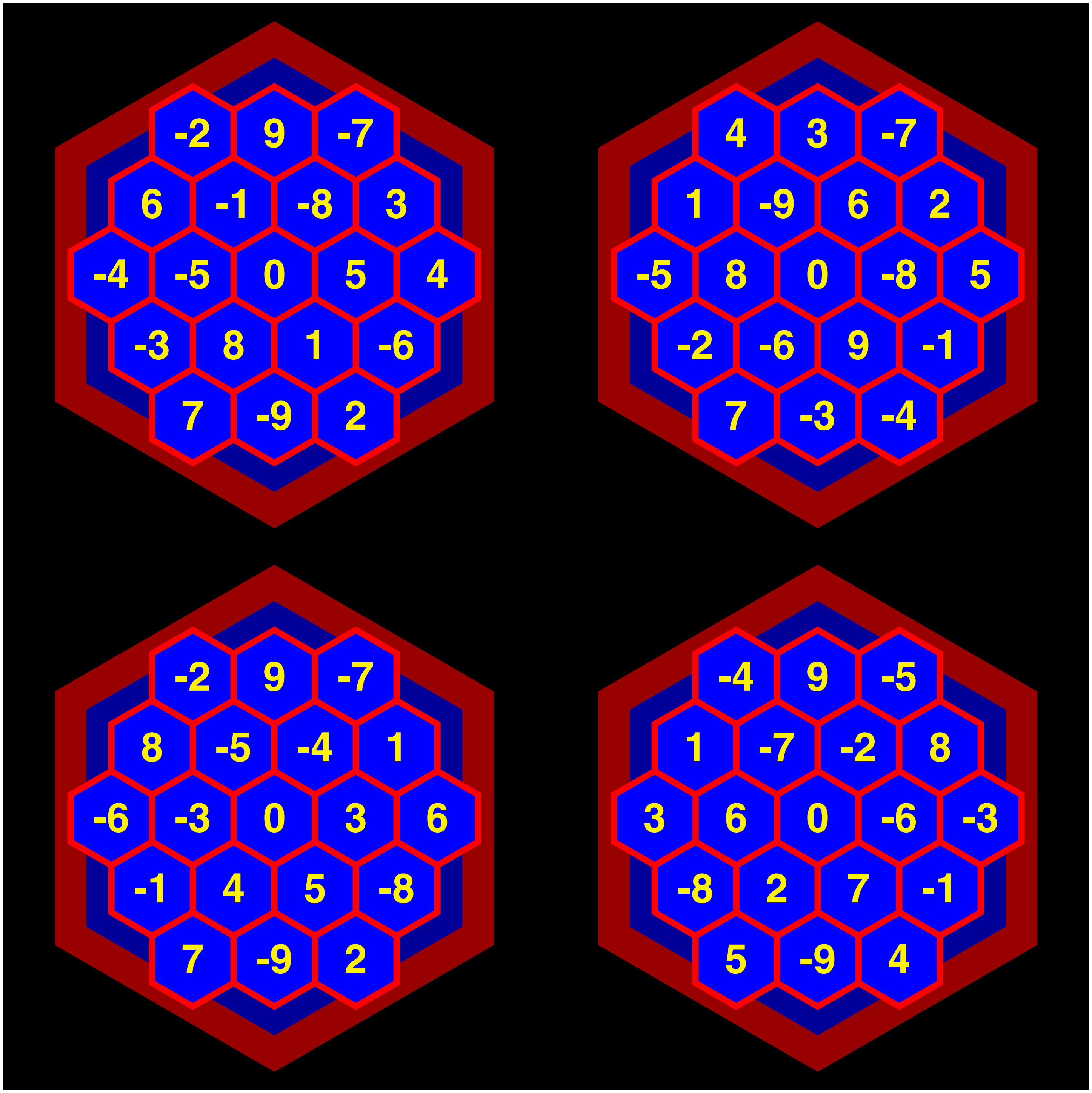
Vital Statistics:
c_R = 3
r = 0
LowNum = -9
HighNum = 9
mNum = 0

ADDS TO 0



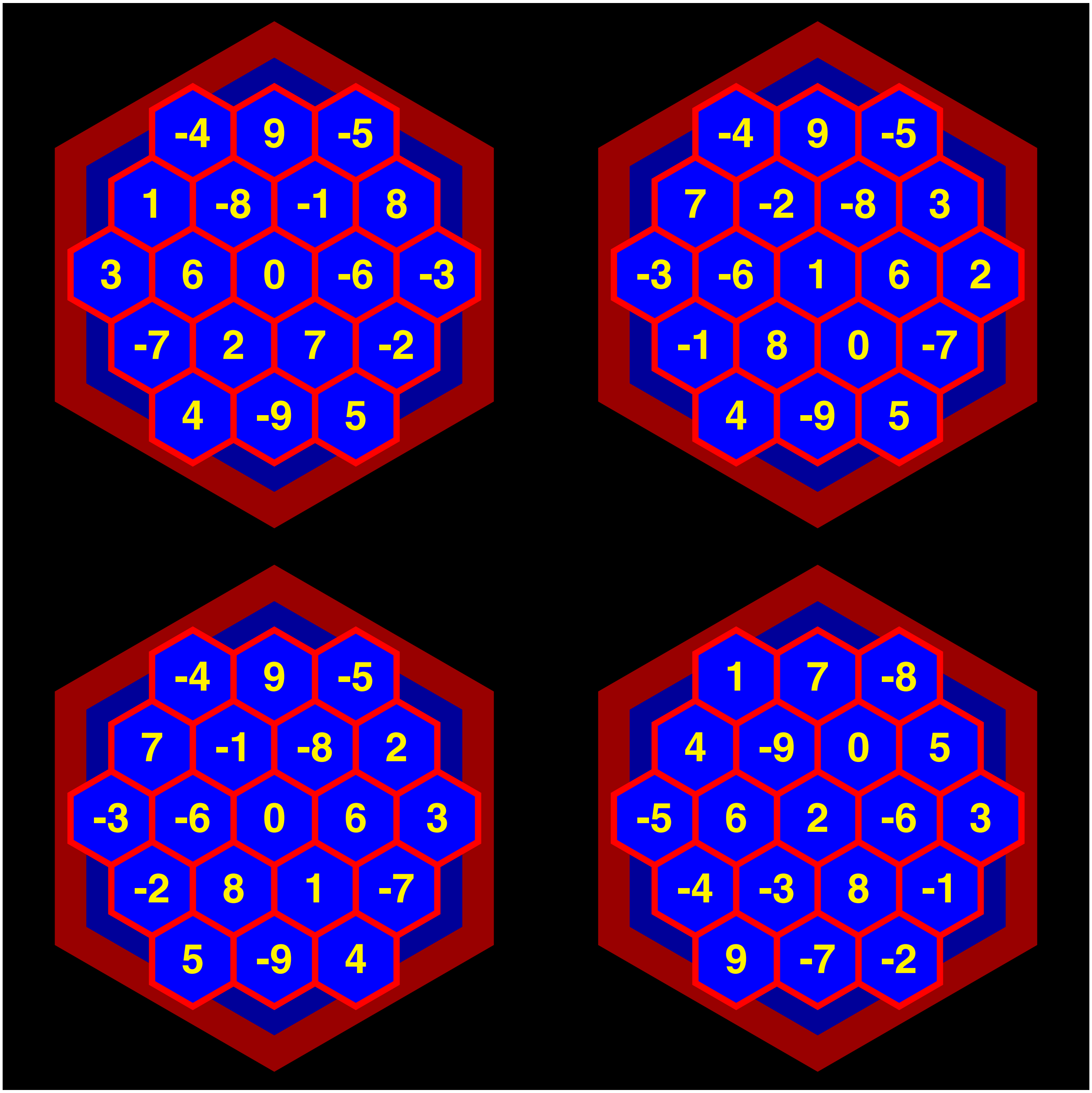
Vital Statistics:
c_R = 3
r = 0
LowNum = -9
HighNum = 9
mNum = 0

ADDS TO 0



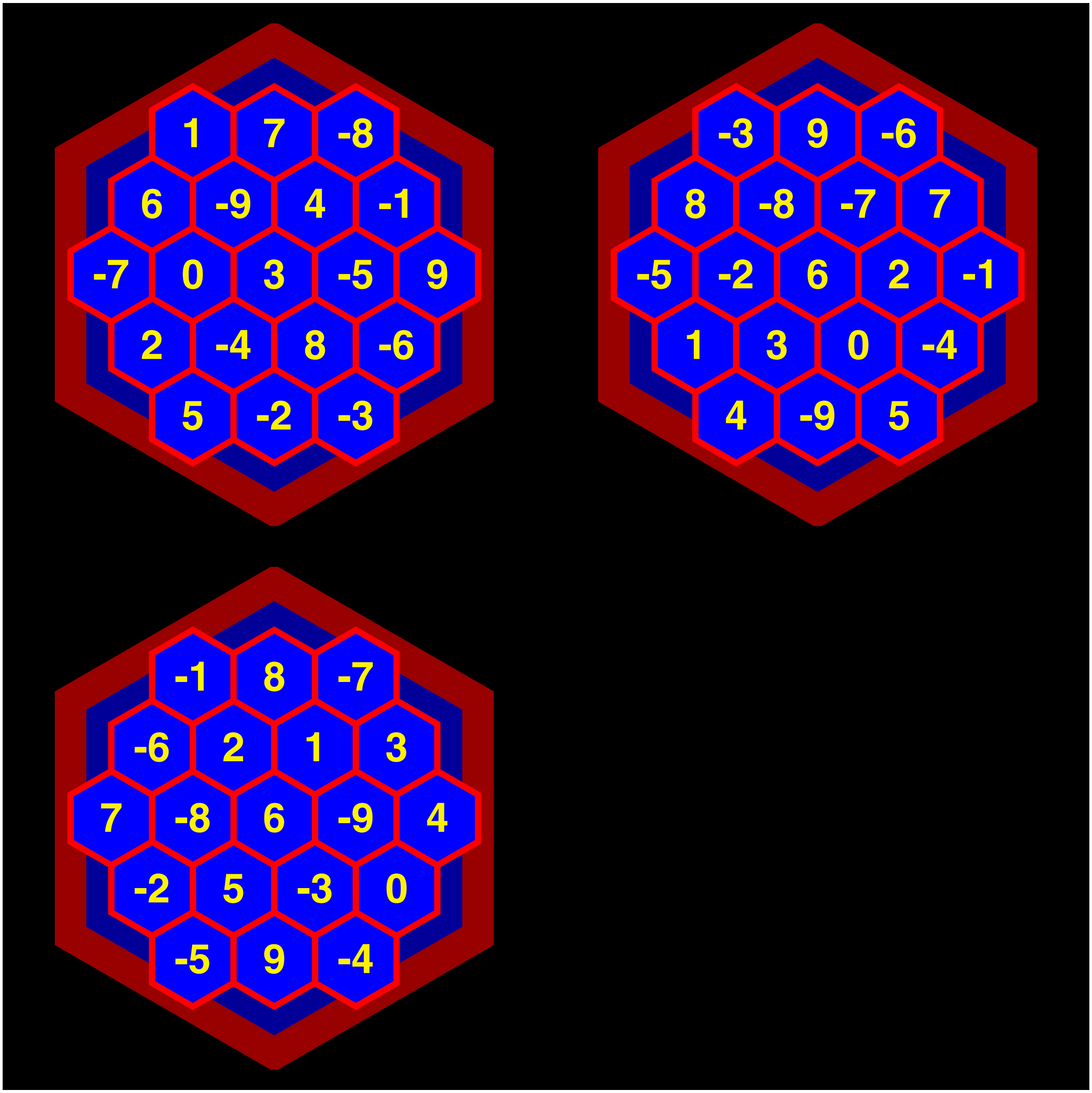
Vital Statistics:
c_R = 3
r = 0
LowNum = -9
HighNum = 9
mNum = 0

ADDS TO 0



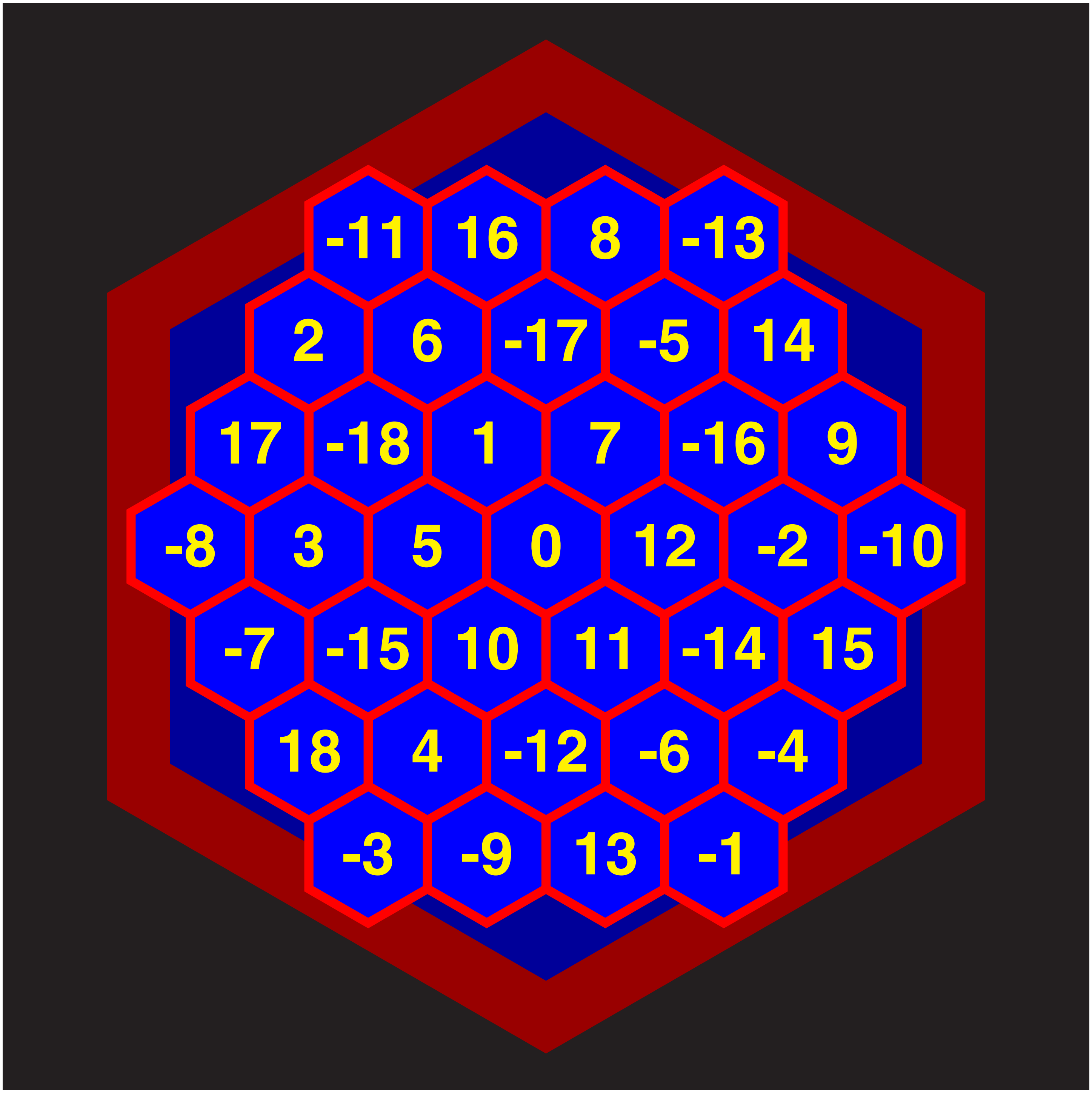
Vital Statistics:
c_R = 3
r = 0
LowNum = -9
HighNum = 9
mNum = 0

ADDS TO 0



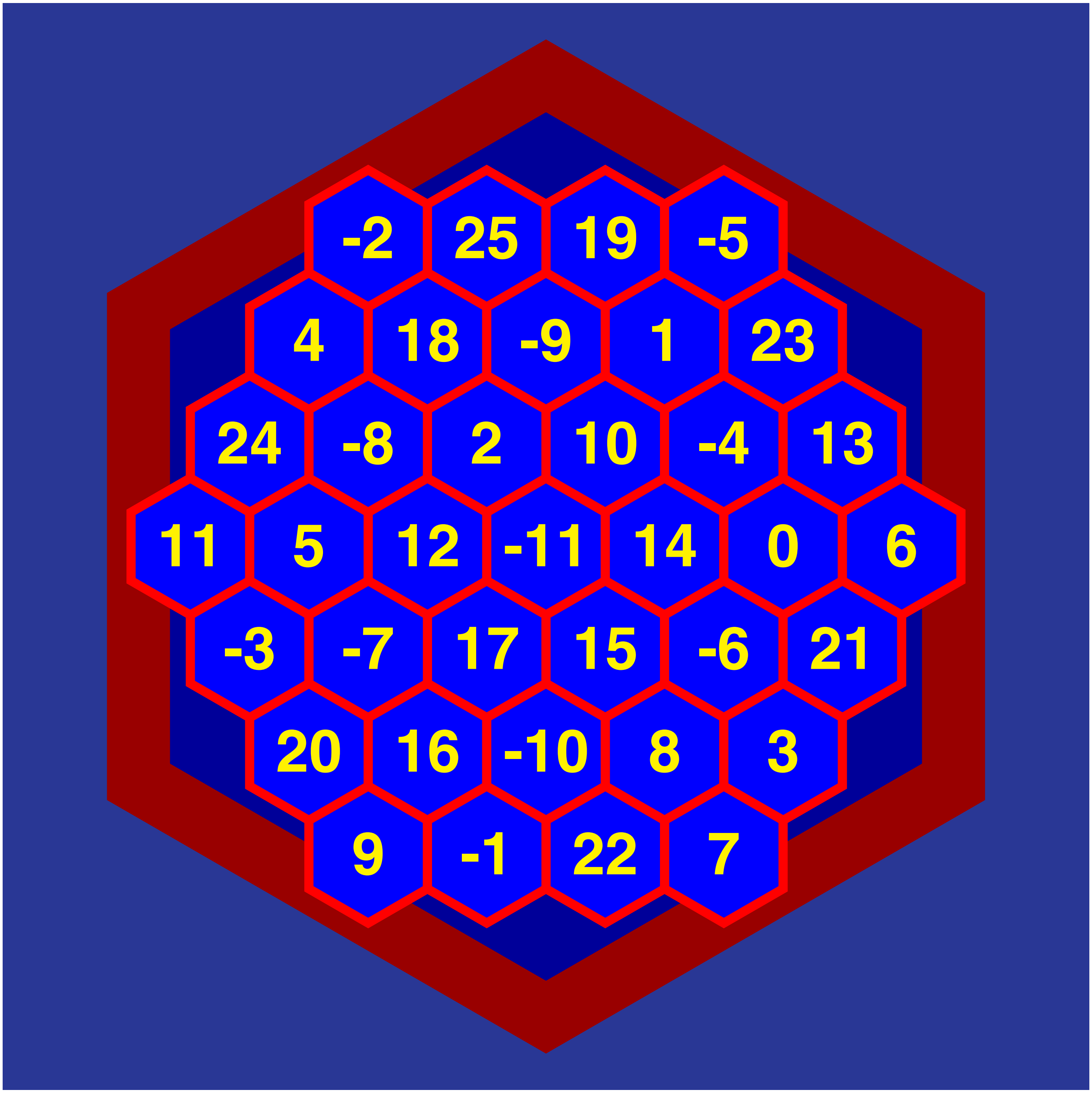
Vital Statistics:
c_R = 3
r = 0
LowNum = -9
HighNum = 9
mNum = 0

4 Ring Magical Hexagon



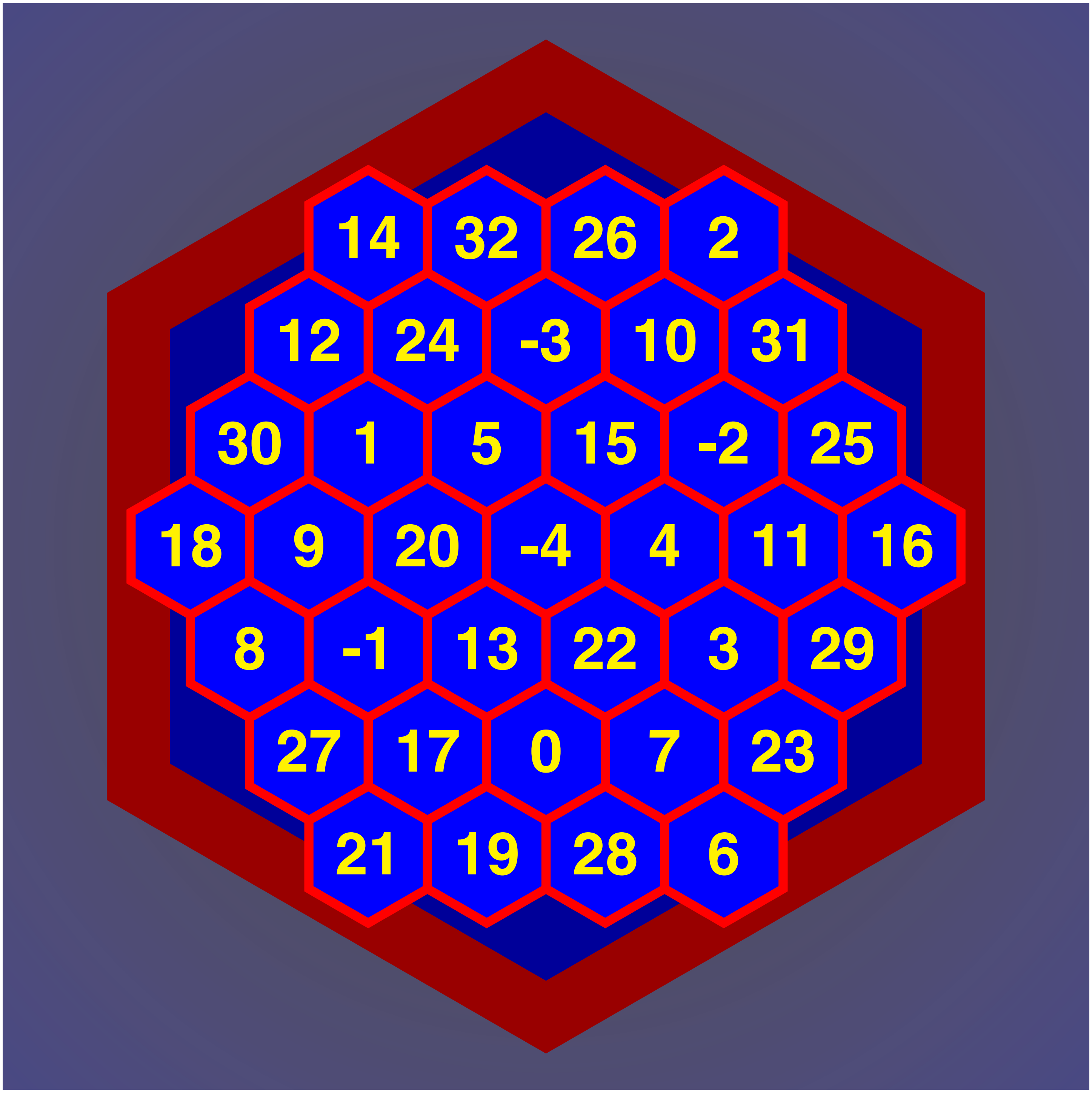
Vital Statistics:
c_R = 4
r = 0
LowNum = -18
HighNum = 18
mNum = 0

4 Ring Magical Hexagon



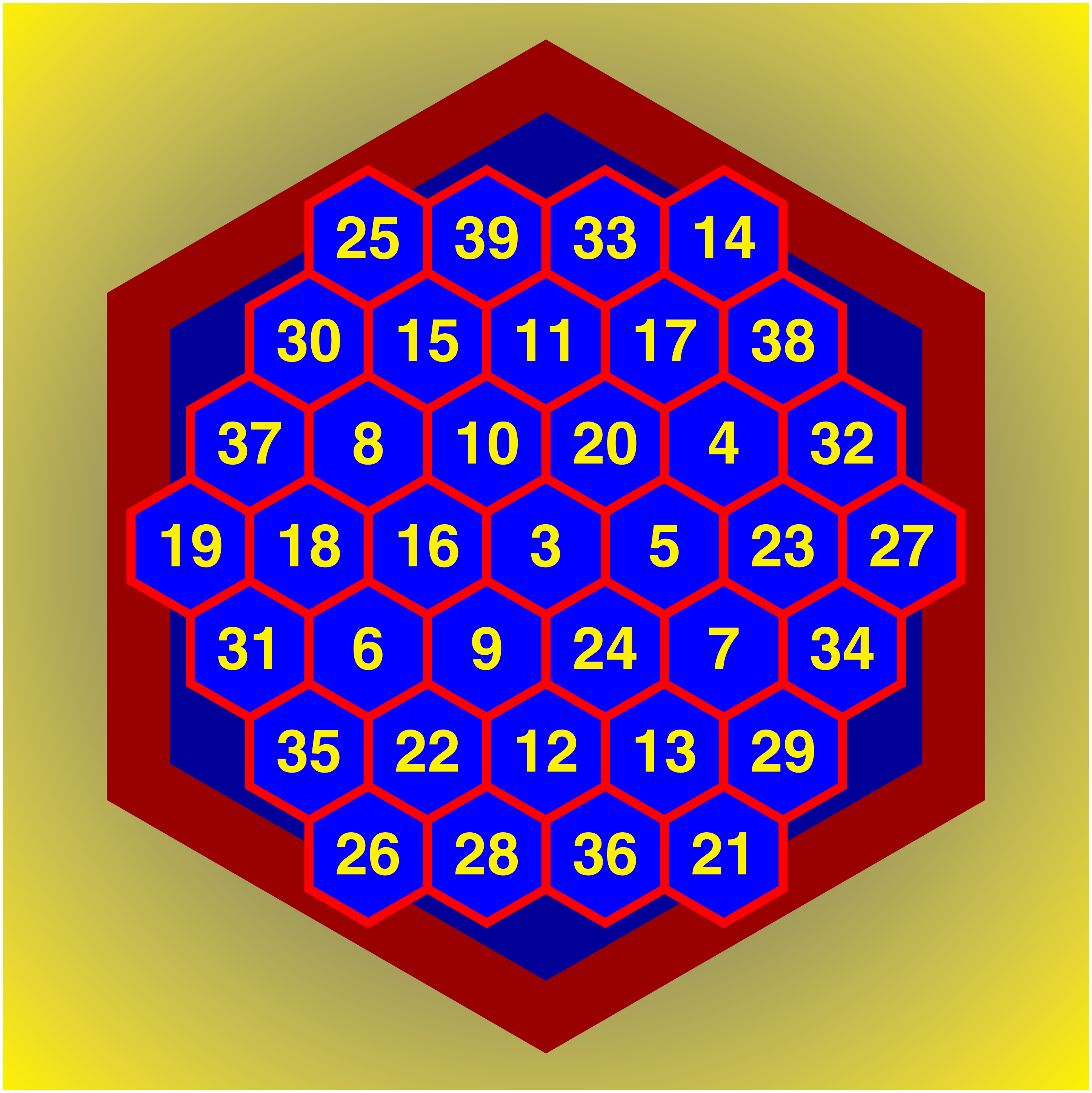
Vital Statistics:
c_R = 4
r = 1
LowNum = -11
HighNum = 25
mNum = 37

4 Ring Magical Hexagon



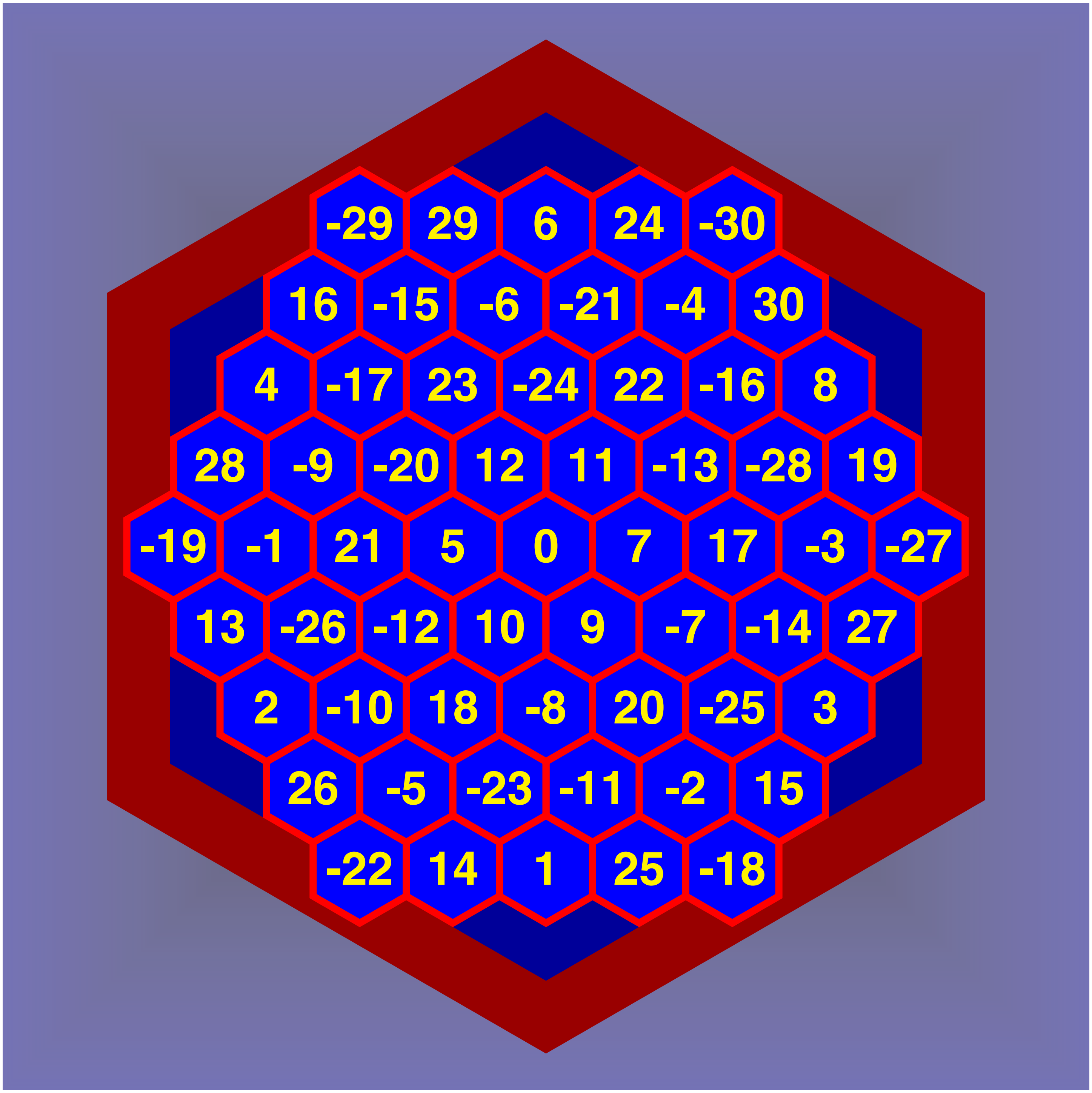
Vital Statistics:
c_R = 4
r = 2
LowNum = -4
HighNum = 32
mNum = 74

4 Ring Magical Hexagon



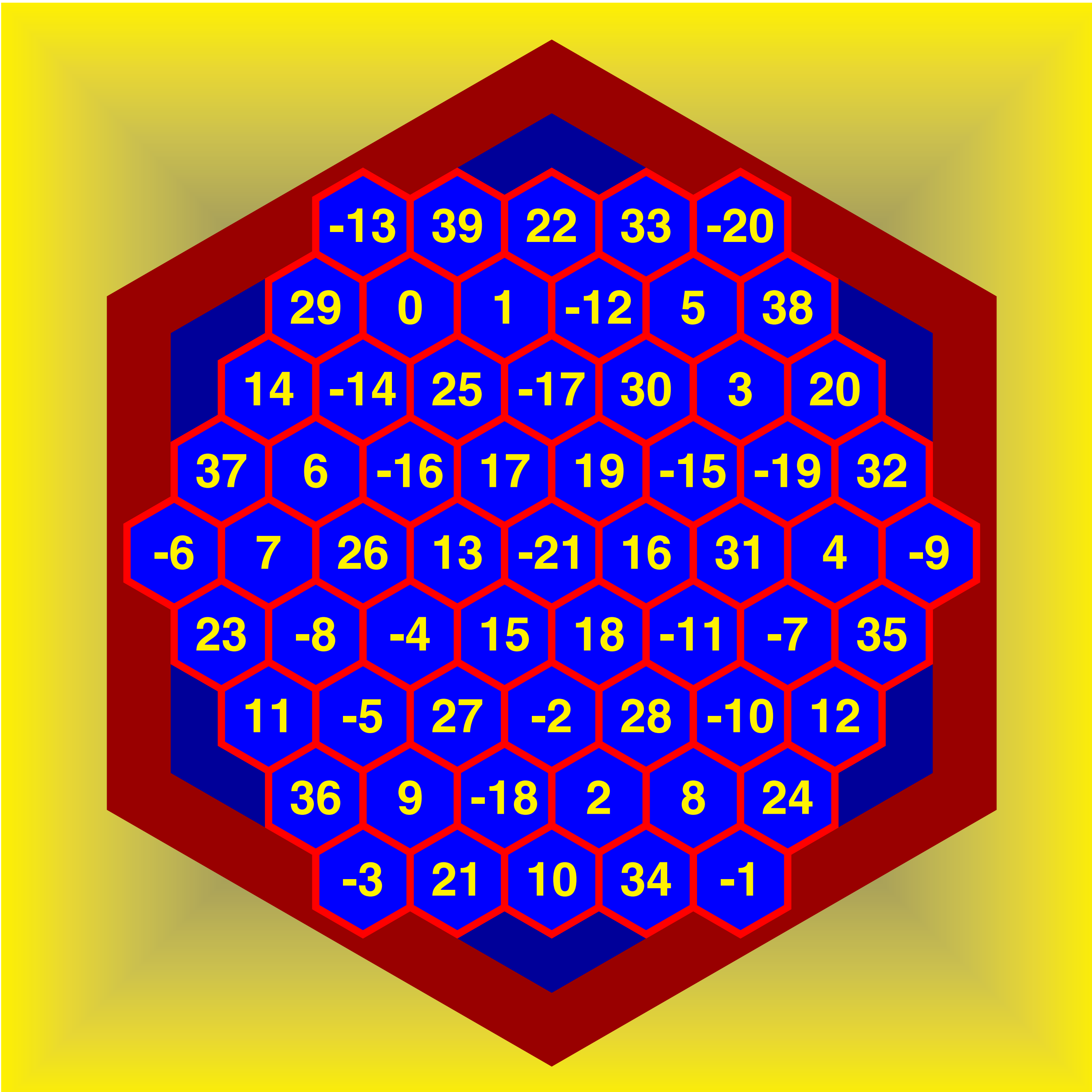
Vital Statistics:
c_R = 4
r = 3
LowNum = 3
HighNum = 39
mNum = 111

5 Ring Magical Hexagon



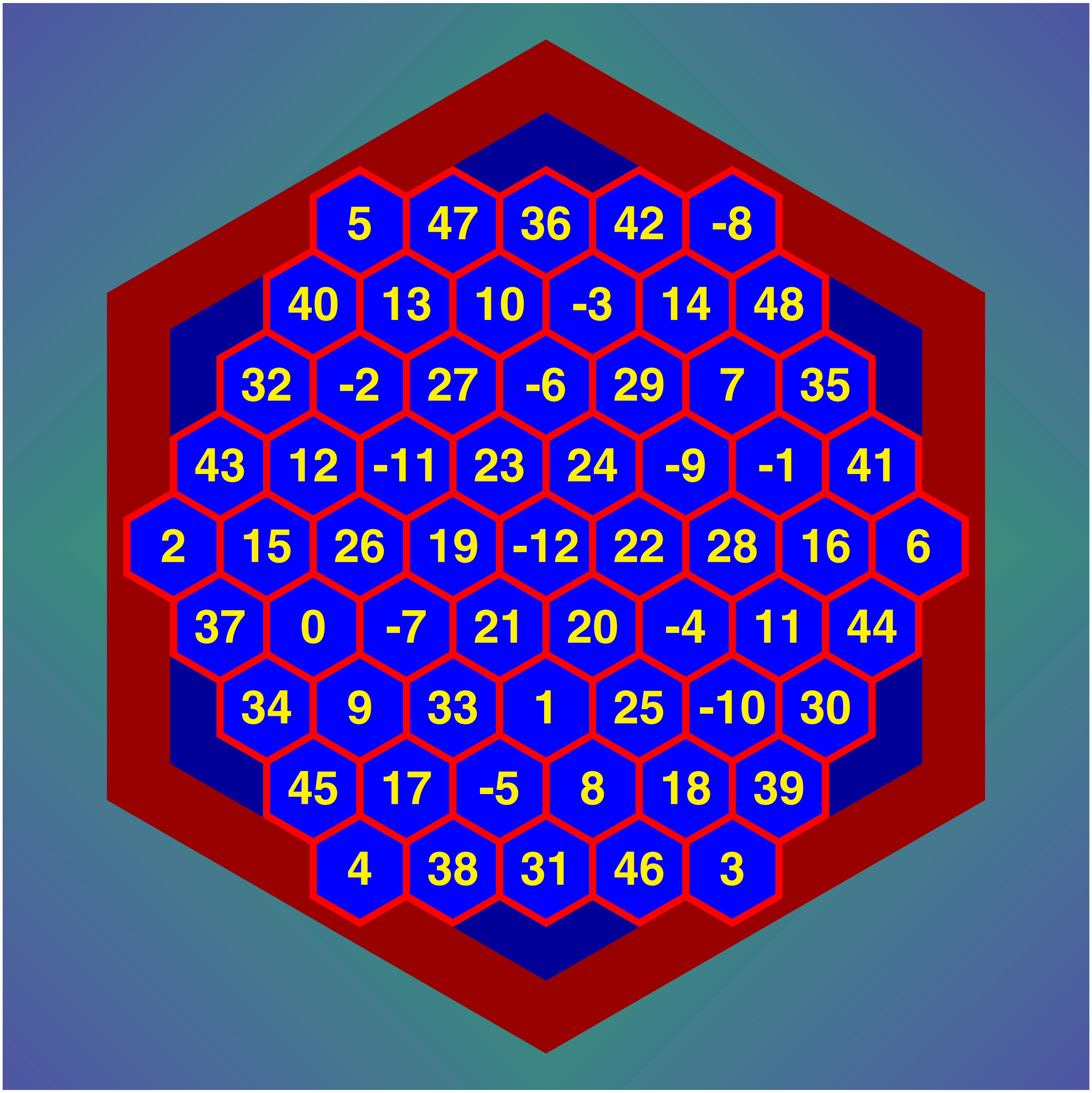
Vital Statistics:
c_R = 5
r = 0
LowNum = -30
HighNum = 30
mNum = 0

5 Ring Magical Hexagon



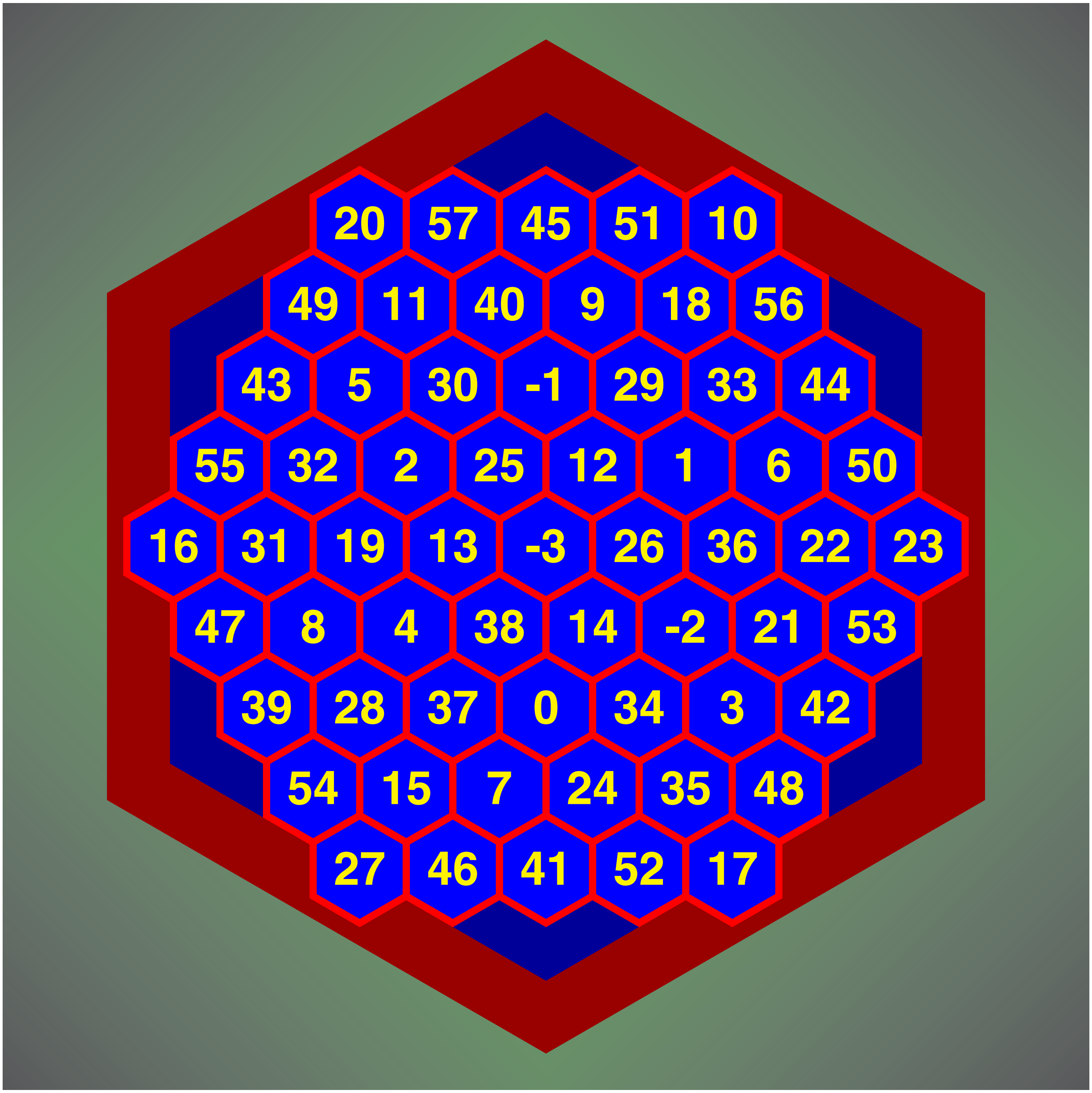
Vital Statistics:
c_R = 5
r = 1
LowNum = -21
HighNum = 39
mNum = 61

5 Ring Magical Hexagon



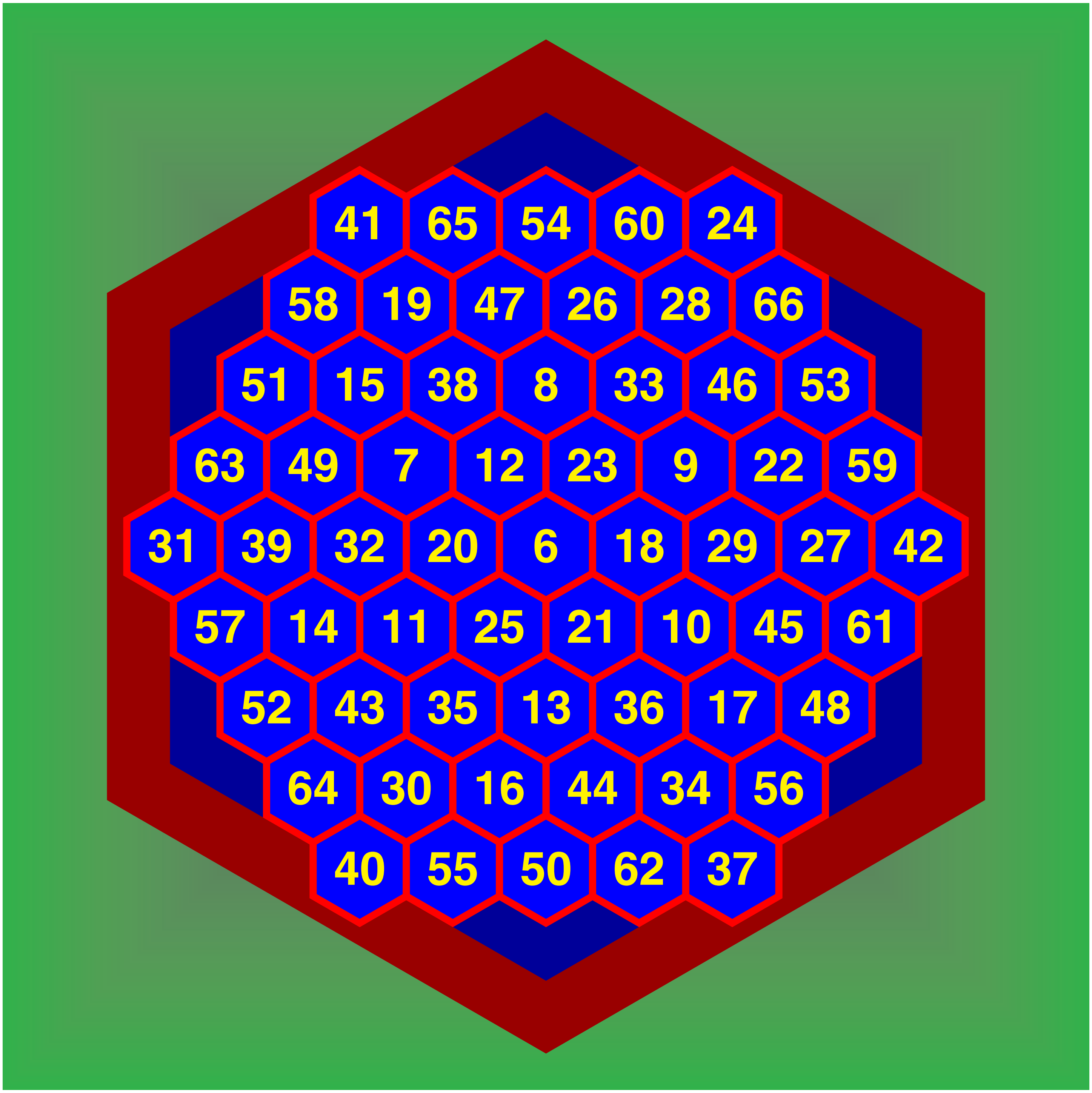
Vital Statistics:
c_R = 5
r = 2
LowNum = -12
HighNum = 48
mNum = 122

5 Ring Magical Hexagon



Vital Statistics:
c_R = 5
r = 3
LowNum = -3
HighNum = 57
mNum = 183

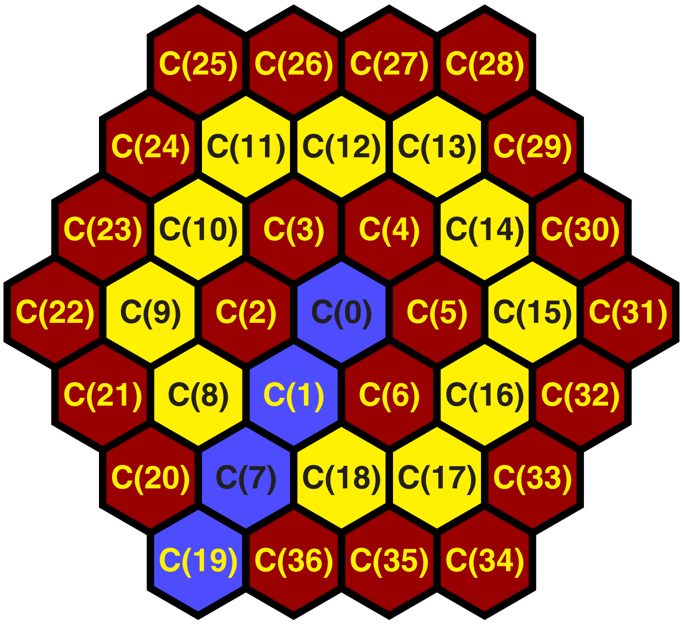
5 Ring Magical Hexagon



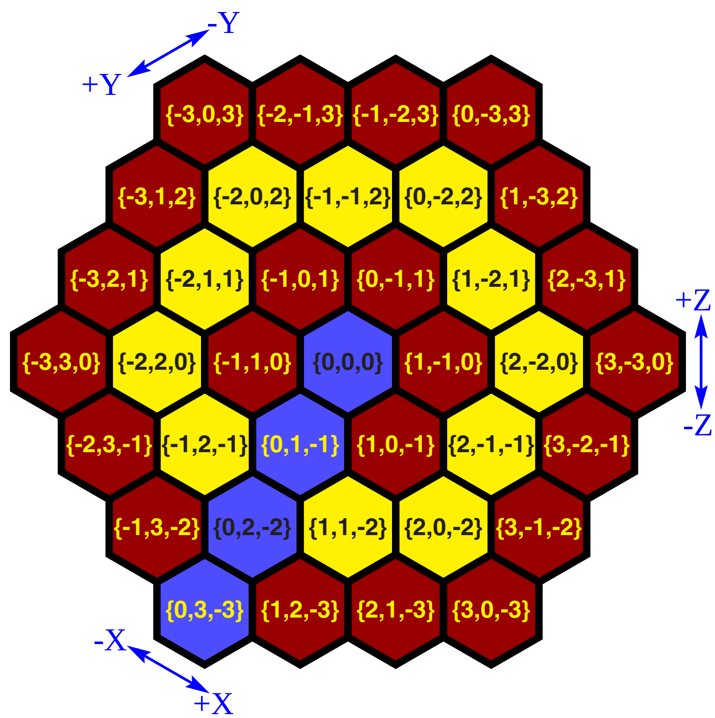
Vital Statistics:
c_R = 5
r = 4
LowNum = 6
HighNum = 66
mNum = 244

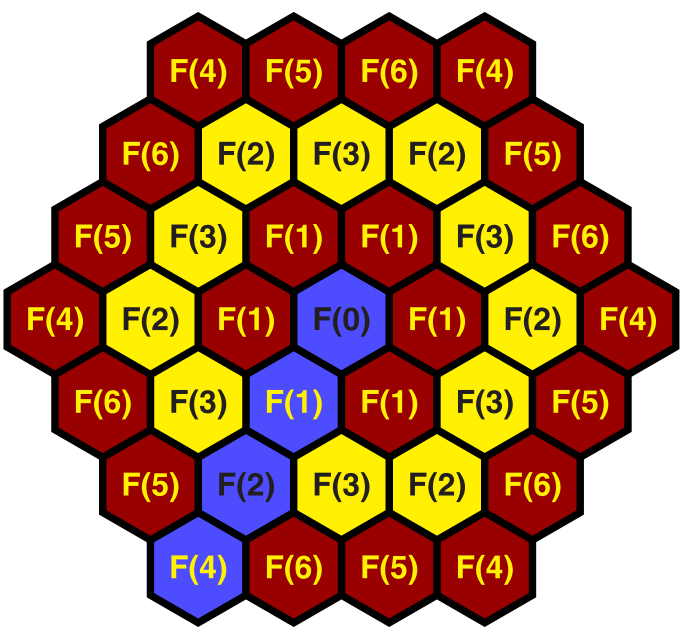
Scratch Pages
The following are sample Items I developed while working on Magical Hexagons.
Explaining them will be part of a future compilation, tentatively titled:
"How To Program Magical Hexagons with VB.Net"

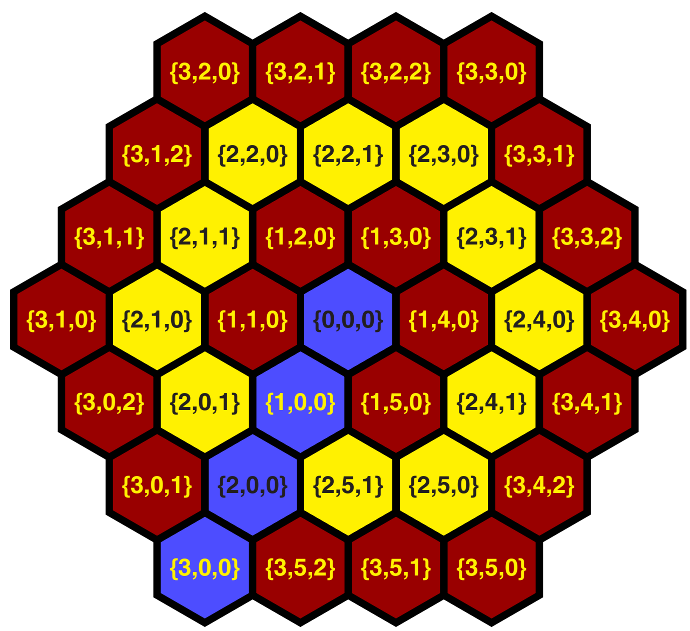
Cell Map



X,Y,Z Map



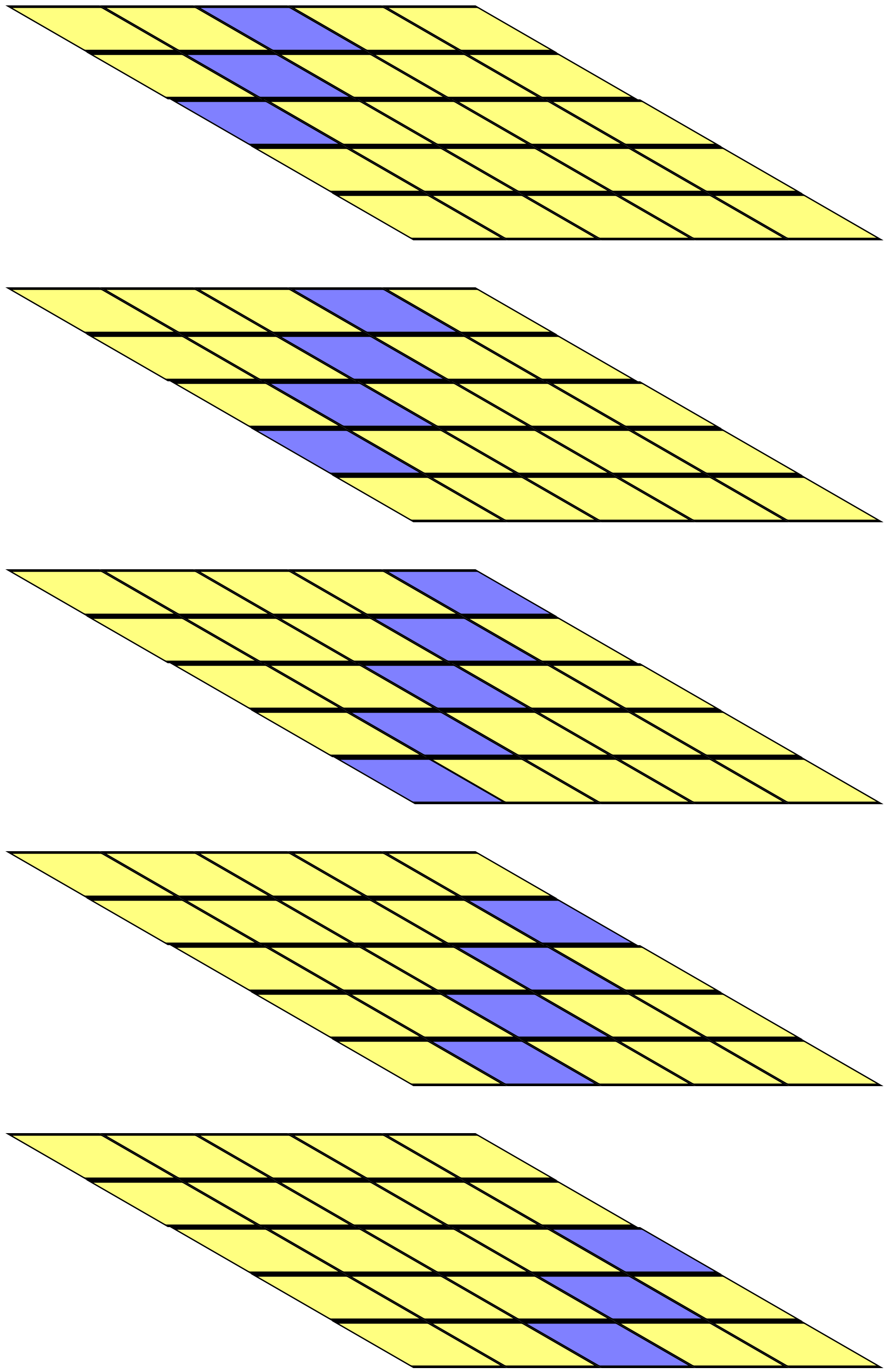




Family Map

Ring, Sector, Offset Map

X+Y+Z = 0, so any 2 of X,Y,Z is sufficient to locate a Cell
A Family consists of 6 Rotationally Equivalent Cells
A Family Index in Combination with a Sector Coordinate will locate an individual Cell.



A Hexagonal Structure can be considered a cross sectioning of a Cube with c_Rows on a side.

If the central cell is considered {0,0,0}, The X, Y, Z Map can be directly applied.

Furthur, A Hexagonal Structure can be “Flattened” Into a 5x5 Square by mapping any two of the three X,Y,Z Coordinates.

3 Ring Sample Equations

Cell Equations:

$C\{0\} = F\{0\}$
$C\{2\} = -C\{1\} - C\{13\} - C\{15\} - F\{0\} + T$
$C\{3\} = C\{1\} + C\{13\} - C\{17\}$
$C\{4\} = -C\{1\} + C\{9\} + C\{11\} + C\{15\} + C\{17\} - T$
$C\{5\} = C\{1\} - C\{9\} + C\{13\}$
$C\{6\} = -C\{1\} - C\{11\} - C\{13\} - F\{0\} + T$
$C\{7\} = -C\{9\} - C\{11\} - C\{13\} - C\{15\} - C\{17\} - F\{0\} + 2T$
$C\{8\} = C\{11\} + C\{13\} + C\{15\} + C\{17\} + F\{0\} - T$
$C\{10\} = -C\{9\} - C\{11\} + T$
$C\{12\} = -C\{11\} - C\{13\} + T$
$C\{14\} = -C\{13\} - C\{15\} + T$
$C\{16\} = -C\{15\} - C\{17\} + T$
$C\{18\} = C\{9\} + C\{11\} + C\{13\} + C\{15\} + F\{0\} - T$

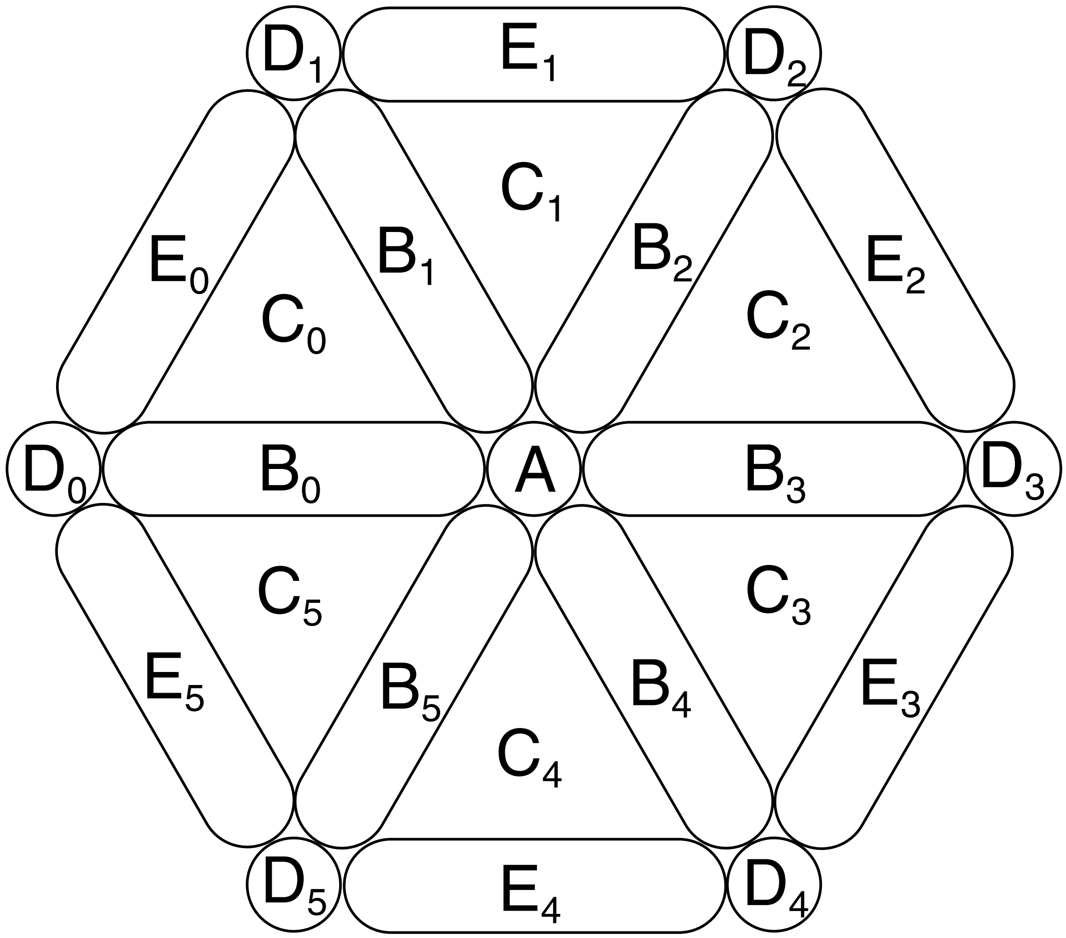
Family Equations:
$F\{1\} = -2F\{0\} + T$
$F\{2\} = -F\{0\} + 2T$
$F\{3\} = 2F\{0\} + 2T$

4 Ring Sample Equations

$C\{0\} = F\{0\}$
$C\{1\} = -C\{4\}+C\{9\}+C\{11\}+C\{15\}+C\{17\}+C\{22\}+C\{25\}+C\{31\}+C\{34\}-F\{4\}$
$C\{2\} = C\{4\}-C\{9\}-C\{11\}-C\{13\}-2C\{15\}-C\{17\}+C\{23\}+C\{25\}+C\{26\}+C\{28\}-C\{31\}-C\{32\}-C\{34\}-C\{35\}-F\{0\}+T$
$C\{3\} = -C\{4\}+C\{9\}+C\{11\}+C\{13\}+C\{15\}+C\{26\}+C\{28\}+2C\{29\}+2C\{31\}+2C\{32\}+C\{34\}+C\{35\}+F\{4\}+F\{6\}-6T$
$C\{5\} = -C\{4\}+C\{11\}+C\{13\}+C\{15\}+C\{17\}-C\{22\}-C\{23\}-C\{25\}-C\{26\}-C\{28\}+C\{32\}+C\{34\}+C\{35\}$
$C\{6\} = C\{4\}-C\{9\}-2C\{11\}-C\{13\}-C\{15\}-C\{17\}-C\{25\}-C\{26\}-C\{28\}-2C\{29\}-2C\{31\}-2C\{32\}-2C\{34\}-C\{35\}$
$-F\{4\}-F\{6\}-F\{0\}+7T$
$C\{7\} = -C\{9\}-C\{11\}-C\{13\}-C\{15\}-C\{17\}-F\{0\}+T$
$C\{8\} = C\{11\}+C\{13\}+C\{15\}+C\{17\}-C\{22\}-C\{23\}-C\{25\}-C\{28\}-C\{31\}+C\{35\}+F\{4\}+F\{0\}-T$
$C\{10\} = -C\{9\}-C\{11\}-C\{23\}-C\{25\}-2C\{26\}-C\{28\}-C\{29\}-C\{31\}-C\{32\}-C\{34\}-C\{35\}-F\{4\}-F\{6\}+6T$
$C\{12\} = -C\{11\}-C\{13\}+C\{22\}+C\{23\}+C\{25\}-C\{29\}$
$C\{14\} = -C\{13\}-C\{15\}+C\{25\}+C\{26\}+C\{28\}-C\{32\}$
$C\{16\} = -C\{15\}-C\{17\}+C\{28\}+C\{29\}+C\{31\}-C\{35\}$
$C\{18\} = C\{9\}+C\{11\}+C\{13\}+C\{15\}+C\{23\}+C\{26\}+C\{29\}+C\{31\}+2C\{32\}+C\{34\}+C\{35\}+2F\{4\}+F\{6\}+F\{0\}-7T$
$C\{19\} = -C\{22\}-C\{25\}-C\{28\}-C\{31\}-C\{34\}+F\{4\}$
$C\{20\} = -C\{23\}-C\{26\}-C\{29\}-C\{32\}-C\{35\}-2F\{4\}-F\{6\}+6T$
$C\{21\} = C\{23\}+C\{25\}+C\{26\}+C\{28\}+C\{29\}+C\{31\}+C\{32\}+C\{34\}+C\{35\}+F\{4\}+F\{6\}-5T$
$C\{24\} = -C\{22\}-C\{23\}-C\{25\}+T$
$C\{27\} = -C\{25\}-C\{26\}-C\{28\}+T$
$C\{30\} = -C\{28\}-C\{29\}-C\{31\}+T$
$C\{33\} = -C\{31\}-C\{32\}-C\{34\}+T$
$C\{36\} = C\{22\}+C\{25\}+C\{28\}+C\{31\}-C\{35\}-F\{4\}+T$

Family Equations:

$F\{1\} = -F\{4\}-2*F\{0\}+2*T$
$F\{2\} = -F\{0\}+T$
$F\{3\} = 2*F\{4\}+2*F\{0\}-2*T$
$F\{5\} = -2*F\{4\}-F\{6\}+6*T$



All Magical Hexagons can be partitioned off into the above Groupings.
The Following Properties exist, { n + k is modulo 6 }

$$E_0 + E_2 + E_4 = E_1 + E_3 + E_5$$

$$C_0 + C_2 + C_4 = C_1 + C_3 + C_5$$

En = A + B(n+3) + B(n+4) + C(n+3) - Cn

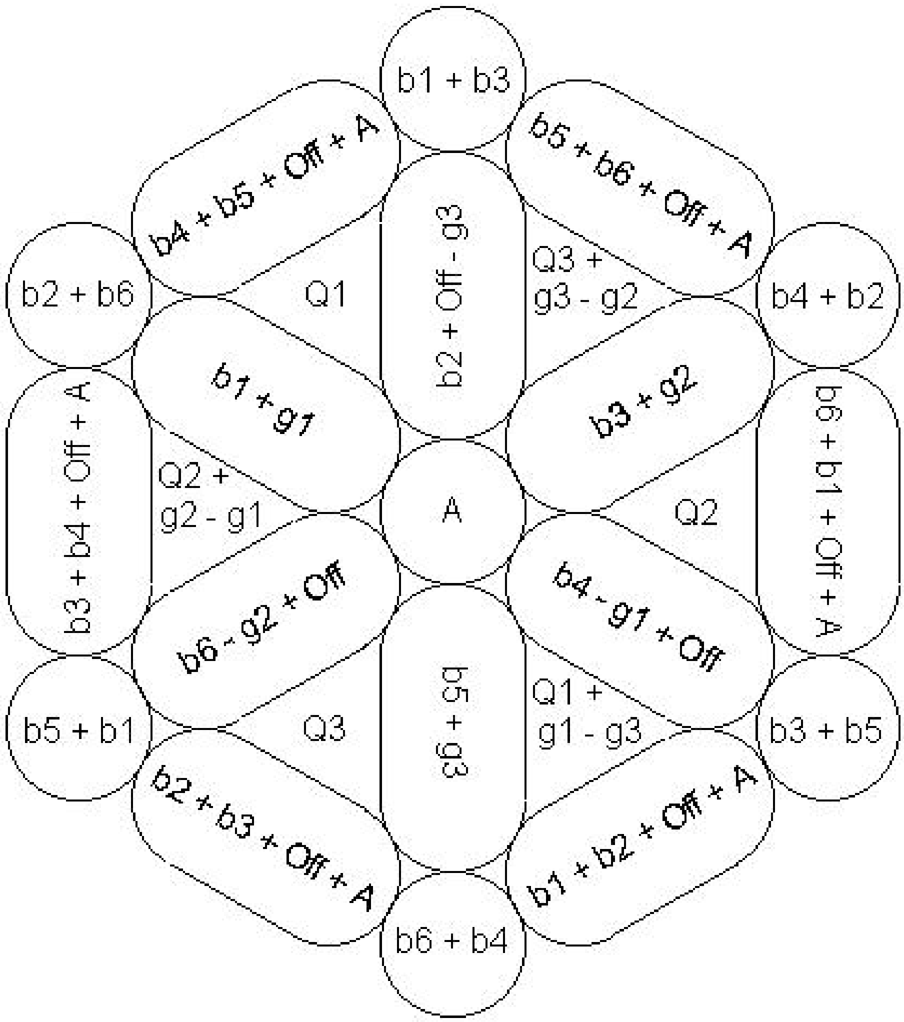
For all Even n, Some s Satisfies:

$$Dn = \{B(n+1) + B(n+5)\} - (1/3)[\{C(n+1)+C(n+4)\} + 2\{C(n+2)+C(n+3)\}] + sA$$

For all Odd n, Some t Satisfies:

$$Dn = \{B(n+1) + B(n+5)\} - (1/3)[\{C(n+1)+C(n+4)\} + 2\{C(n+2)+C(n+3)\}] + tA$$

July 25th,2002:
Just did this:



Where:
$\mathrm{Sum}(b) + A + \mathrm{Off} = T_1$
if the corners were known with values h1, h2, h3, h4, h5, and h6, then

b1 = (h2+h6-h4)/2	b2 = (h1+h3-h5)/2
b3 = (h4+h2-h6)/2	b4 = (h3+h5-h1)/2
b5 = (h6+h4-h2)/2	b6 = (h5+h1-h3)/2

Also,
$Q1+Q2+Q3=(m-1)T_1 - 2*\mathrm{Off} - A$